

# Zoom-Dependent Camera Calibration in Digital Close-Range Photogrammetry

C.S. Fraser and S. Al-Ajlouni

## Abstract

One of the well-known constraints applying to the adoption of consumer-grade digital cameras for photogrammetric measurement is the requirement to record imagery at fixed zoom and focus settings. The camera is then calibrated for the lens setting employed. This requirement arises because calibration parameters vary significantly with zoom/focus setting. In this paper, a zoom-dependent calibration process is proposed whereby the image coordinate correction model for interior orientation and lens distortion is expressed as a function of the focal length written to the EXIF header of the image file. The proposed approach frees the practitioner from the requirement to utilize fixed zoom/focus settings for the images forming the photogrammetric network. Following a review of the behavior of camera calibration parameters with varying zoom settings, an account of the newly developed zoom-dependent calibration model is presented. Experimental results of its application to four digital cameras are analysed. These show that the proposed approach is suited to numerous applications of medium-accuracy, digital, close-range photogrammetry.

## Introduction

In order to satisfy the metric requirements of photogrammetric measurement, calibrated cameras are needed. Basically, a calibrated camera is one for which the values of principal distance, principal point offset, and lens distortion as a function of image point location are known. Within the computer vision literature, these parameters of interior orientation and, generally, radial distortion alone are known as intrinsic parameters. The determination of calibration parameters in close-range photogrammetry could be said to have dramatically evolved in the early 1970s when laboratory camera calibration gave way to the concept of self-calibration using multi-station photogrammetric bundle adjustment (Brown, 1971 and 1974; Kenefick *et al.*, 1972). Accompanying the introduction of self-calibration was the removal of a number of constraints upon the design of metric cameras: focusable lenses could now be deemed metric if the focal setting could be held fixed, and changes in interior orientation parameters were of no consequence so long as the parameters remained invariant throughout the course of a photogrammetric survey.

Self-calibration remains the norm today in multi-image, close-range photogrammetry. However, with the increasing adoption of consumer-grade digital cameras for applications as diverse as traffic accident reconstruction, process plant documentation, and cultural heritage recording and modeling, constraints upon image acquisition geometry imposed

to facilitate self-calibration can become an impediment in many operational situations. For example, the traditional *physical model* of eight calibration parameters of principal distance  $c$ ; principal point coordinates  $x_p$ ,  $y_p$ , radial distortion parameters  $K_1$ ,  $K_2$ , and  $K_3$ , and decentering distortion coefficients  $P_1$ ,  $P_2$  (e.g., Brown, 1971; Fraser, 1997) are focal setting dependent. Thus, to ensure invariance of these parameters, the user conducting a photogrammetric survey with a consumer-grade digital camera must adopt both a constant zoom setting and fixed focus. The former requirement is relatively easy to accommodate, but the latter may not be so simple because manual focus controls may not even be present, all images being auto-focussed.

It is of course possible to conduct a multi-focal setting or multi-zoom self-calibration, although there is the requirement that sufficient images at a particular focus/zoom setting are available in an appropriate geometric configuration in order to recover the interior orientation and lens distortion parameters. The multi-focal setting self-calibration can also incorporate constraints to ensure that the linear variations of radial distortion that occur with changing focus are rigorously modelled (Fraser, 1980). Such distortion variations have long been known to be significant for large-format lenses, especially at close focal settings. However, given the fact that the variation diminishes significantly at focussed distances beyond about 15 focal lengths (Fryer and Brown, 1986; Fraser and Shortis, 1990), the problem could be anticipated to be of much less significance for consumer-grade digital cameras which invariably have maximum focal lengths of less than 30 mm, rendering almost all practical photogrammetric imagery infinity focussed. For example, a commonly encountered zoom lens in off-the-shelf digital cameras is one incorporating 3 $\times$  optical zoom, from 7 to 21 mm focal length.

Given the constraints related to self-calibration of cameras with auto-focus zoom lenses, it would be desirable to have at the disposal of the close-range photogrammetry practitioner a *zoom-dependent* calibration function whereby the interior orientation and distortion parameters would be known for any focus or zoom setting, and for all practical purposes, for the zoom setting alone. This paper describes such a zoom-dependent (referred to as Z-D) camera calibration process and discusses its metric implications. A prerequisite for the technique developed is that the focal length (not necessarily the photogrammetric principal distance) for every individual image is written to the Exchange Image Format (EXIF) header of the recorded digital image, which is usually a JPEG file. The operational scenario is that once

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Department of Geomatics, University of Melbourne, Victoria  
3010, Australia (c.fraser@unimelb.edu.au; sajlouni@sunrise.  
sli.unimelb.edu.au).

the Z-D calibration function is determined, the images of the photogrammetric network can be recorded at whatever zoom setting is most appropriate at each camera station. The resulting bundle adjustment must accommodate as many principal distance values as images and self-calibration is precluded, though it is also unnecessary.

In the following sections we first review the behavior of camera calibration parameters in the presence of varying zoom settings. For the case of variation of radial distortion with focussing, a physical model is applicable (Brown, 1971), whereas for the remaining calibration parameters empirical models must be relied upon. In order to assess the degree of success of the proposed Z-D calibration process, we consider in this investigation four different consumer-grade digital cameras which incorporate no special features to accommodate the traditional requirements of close-range photogrammetry. With the development described being directed towards off-the-shelf cameras, it is clear that the Z-D calibration concept is unlikely to be suited to photogrammetric surveys seeking the highest possible accuracy. In these instances, self-calibration will remain the norm. Instead, we concentrate upon facilitating moderate accuracy object point determination with maximum operational flexibility.

### Initial Assumptions

In order for the Z-D calibration process to assume a reasonable degree of validity, we need to operate with some basic initial assumptions. These can be summarized as follows:

- While shortcomings in accurately modelling calibration parameters as a function of zoom setting are of concern, they are secondary to the degree to which accuracy in 3D point determination within a photogrammetric triangulation is preserved using a single-function, Z-D image coordinate correction as opposed to rigorous calibration. Projective compensation is the mechanism by which systematic errors introduced from one source, e.g., incorrect interior orientation, are compensated using another subset of parameter, e.g., exterior orientation elements. A measure of projective compensation is likely to mitigate somewhat small errors in calibration parameters, be they from temporal influences or from limitations in empirically modelling variation functions.
- Variation of lens distortion with changing focus is of minor consequence in comparison to variation of distortion with zoom setting. The correction functions derived by Brown (1971) have proven very useful in high-precision industrial photogrammetry where cameras of large format are preferred. However, for small format lenses employed in low-cost digital cameras, the experience of the authors and others is that radial lens distortion typically varies by an insignificant amount with changing focus, especially for mid- to far-field focus. As it happens, it is generally not practical to consider distortion modelling with changing focus simply because (a) the associated variations in principal distance are of such a small magnitude with small format lenses as to defy reliable determination using self-calibration, and (b) most zoom cameras write a single focal length value to the EXIF header which gives the zoom setting, but does not reflect changes in focus. We return briefly to this topic later in the paper.
- As the Z-D calibration process will entail interpolation from known calibration values at given zoom settings, it is imperative that the means to comprehensively calibrate the camera at the selected settings is available. Logically, self-calibration would be applied and for practical reasons, we would assume that the traditional photogrammetric self-calibration process involving a single zoom and focus setting would be fully automatic, through the use of arrays of coded targets for example (Fraser and Hanley, 2004). Better still would be a simultaneous self-calibration that incorporated all image networks from all zoom settings, with the bundle adjustment having a separate set of additional parameters (APs) for each zoom setting, i.e., effectively a multi-camera self-calibration.

- A final initial assumption is that only low to moderate 3D measurement accuracy is being sought, though still mainly from multi-station photogrammetric networks as opposed to simple stereo configurations. Proportional accuracies needed for applications in heritage recording and traffic accident reconstruction are invariably at the 1:1000 level (1 cm accuracy over a 10 m object size), and it is with such accuracy requirements in mind that the Z-D calibration process has been developed.

We now turn our attention to the behavior in the presence of changing zoom settings of the individual parameters within the photogrammetric camera calibration model, namely the principal distance, principal point offsets, and radial and decentering distortion.

### Variation in Calibration Parameter Values with Zoom Setting

#### Principal Distance

The Z-D calibration parameters will be determined as a function of principal distance. Thus, it is important to establish the relationship between principal distance and the focal length recorded for each zoom/focus setting by the camera. As mentioned, the authors' experience with consumer-grade cameras suggests that for all practical purposes, photogrammetric principal distance varies by an insignificant amount with focus for a particular zoom setting. To highlight this, we consider the self-calibration of two cameras at two zoom settings each. Independent determinations of principal distance were made at three focus settings for each zoom value. The first camera was an 8-megapixel Sony Cyber-shot F828 focussed at 0.5 m, 1.5 m, and 3 m for zoom focal lengths of 7.1 and 16 mm. For both zoom settings, the computed photogrammetric principal distance varied by less than 0.03 mm for the three focus settings. The second camera was a Canon PowerShot G1, which also had a 3 $\times$  optical zoom lens. This camera was calibrated at focus settings of 0.3 m, 1 m, and infinity for zoom settings of 7 mm (zoomed fully out) and 14.6 mm. Once again, the variation of principal distance was of insignificantly small magnitude being 0.03 mm for the 7.1 mm zoom and 0.01 mm for the 14.6 mm focal length. Thus, for the Z-D calibration technique, there is only a need to consider the relationship between photogrammetric principal distance and the focal length value written to the EXIF Header.

Shown in Table 1 are the differences between computed principal distance and recorded focal length for multiple zoom settings for the four cameras studied in this paper. The cameras are a 6-megapixel Nikon D100 with an external Nikkor 24 to 85 mm zoom lens, two 3-megapixel Canon PowerShot cameras, a G1 and an S30, and a 2-megapixel Canon IXUS V. In three cases the recorded focal length is that written to the EXIF header, though for the 24 to 85 mm lens on the D100 SLR-type camera the zoom was manually set using marks on the lens barrel. From Table 1 it is apparent that the discrepancy between recorded focal length and the computed principal distance is not a constant value. Moreover, the variation is non-linear, though expressed as a fraction of the absolute focal length it is generally small. Under the practical scenario envisaged for Z-D calibration, some three zoom settings (zoomed in, out, and somewhere in the mid-range) would be used to determine the variation functions for the calibration parameters. Thus, one has little practical option other than to assume either a constant or a linear variation function for the determination of the photogrammetric principal distance,  $c$ , from the recorded zoom focal length,  $f$ . A linear variation function has been adopted in this investigation.

TABLE 1. DIFFERENCES BETWEEN PHOTOGRAMMETRIC PRINCIPAL DISTANCE ( $c$ ) AND RECORDED FOCAL LENGTH ( $f$ ) FOR MULTIPLE ZOOM SETTINGS. UNITS ARE MM

Canon PowerShot S30		Canon PowerShot G1		Canon IXUS V		Nikon D100 with 24 to 85 mm Lens	
Zoom Setting	Difference $c-f$	Zoom Setting	Difference $c-f$	Zoom Setting	Difference $c-f$	Zoom Setting	Difference $c-f$
7.1	0.42	7	0.27	5.4	0.12	24	1.7
8.6	0.35	10.8	-0.10	6.7	-0.06	35	0.8
10.3	0.31	16.8	0.05	8.0	-0.12	50	0.8
12.3	0.32	18.8	0.24	9.3	-0.14	70	2.2
17.5	0.26	21.0	-0.07	10.8	-0.37	85	0.4
21.3	-0.30						

### Principal Point Offset

Shown in Figure 1 are both the values of the principal point offsets  $x_p$  and  $y_p$  for the four cameras and zoom setting considered in Table 1, and best-fit linear variation functions for  $x_p$  and  $y_p$  determined using only the three settings indicated by the solid dots. As could be anticipated, different cameras show very different variation in principal point coordinates with zoom. While a reasonably linear trend is apparent in some cases, suggesting a near constant misalignment angle between the optical axis of the lens and the focal plane, others show non-linear behavior. These characteristics have also been seen in previous investigations (Laebe and Foerstner, 2004; Wiley and Wong, 1995; Noma *et al.*, 2002). Compounding the issue in self-calibration is the high projective coupling between decentering distortion and principal point offset, with it being difficult to fully isolate these two sets of calibration parameters, especially at longer focal lengths. The plots shown in Figure 1 simply reinforce the view that while a non-linear relationship might exist, the only practical way to model principal point coordinates as a function of zoom in the proposed Z-D calibration is through a linear model, especially given that we suggest the routine use of three zoom settings to determine the Z-D image coordinate correction functions.

### Radial Lens Distortion

Based on an original formulation by Magill (1955), Brown (1971) first derived a formula for the correction to image coordinates for the linear variation in radially symmetric, Gaussian lens distortion with changing focus. This model was extended further by Brown to also account for variations of distortion within the photographic field. In the context of Z-D calibration, however, these developments are of limited utility, since we have already indicated that the variation with focus of principal distance, and consequently radial distortion, is insignificantly small for consumer-grade digital cameras. The same is not true, however, when it comes to the variation of radial distortion with zoom setting.

Shown in Figure 2 are the Gaussian radial distortion profiles for the PowerShot S30 and G1 cameras, and for the IXUS V, for various zoom settings. Also shown are the corresponding profiles for the D100 with 24 to 85 mm lens. These profiles are obtained using the well-known expression:

$$dr = K_1 r^3 + K_2 r^5 + K_3 r^7 \quad (1)$$

where  $dr$  is the radial distortion,  $r$  the radial distance and  $K_1$ ,  $K_2$ , and  $K_3$  the coefficients of symmetric radial distortion. The plots in Figure 2 are derived using self-calibration, and the profiles are plotted only to the maximum radial distance encountered in the self-calibration, i.e., extrapolated portions of the curves to the corner of the image format are not shown.

The first feature of note in Figure 2 is that the radial distortion variation is non-linear, and that maximum distortion occurs at minimum focal length, even in cases where a zero crossing occurs. Not obvious from the figure, however, is another important feature, namely that the coefficient  $K_1$  describing the cubic component of radial distortion is monotonically decreasing with increasing zoom, a fact that follows from the previous observation since  $K_1$  is far and away the dominant coefficient. This characteristic has been observed in zoom lenses by others (Laebe and Foerstner, 2004; Wiley and Wong, 1995; Burner, 1995; Fryer, 1986), as has the fact that, except for the shorter focal lengths of the zoom range, the coefficients  $K_2$  and  $K_3$  can be suppressed for all practical purposes.

Noting that the combined contribution to lens distortion of the terms in  $K_2$  and  $K_3$  is generally quite small, the focus of modelling variation in radial distortion with zoom setting can be placed on the behaviour of  $K_1$ . Shown in Figure 3 are plots of the variation in  $K_1$  with focal length for the four cameras under consideration. We have found that the non-linear monotonic decrease in  $K_1$  with increasing zoom is very amenable to modelling by an empirical function of the form:

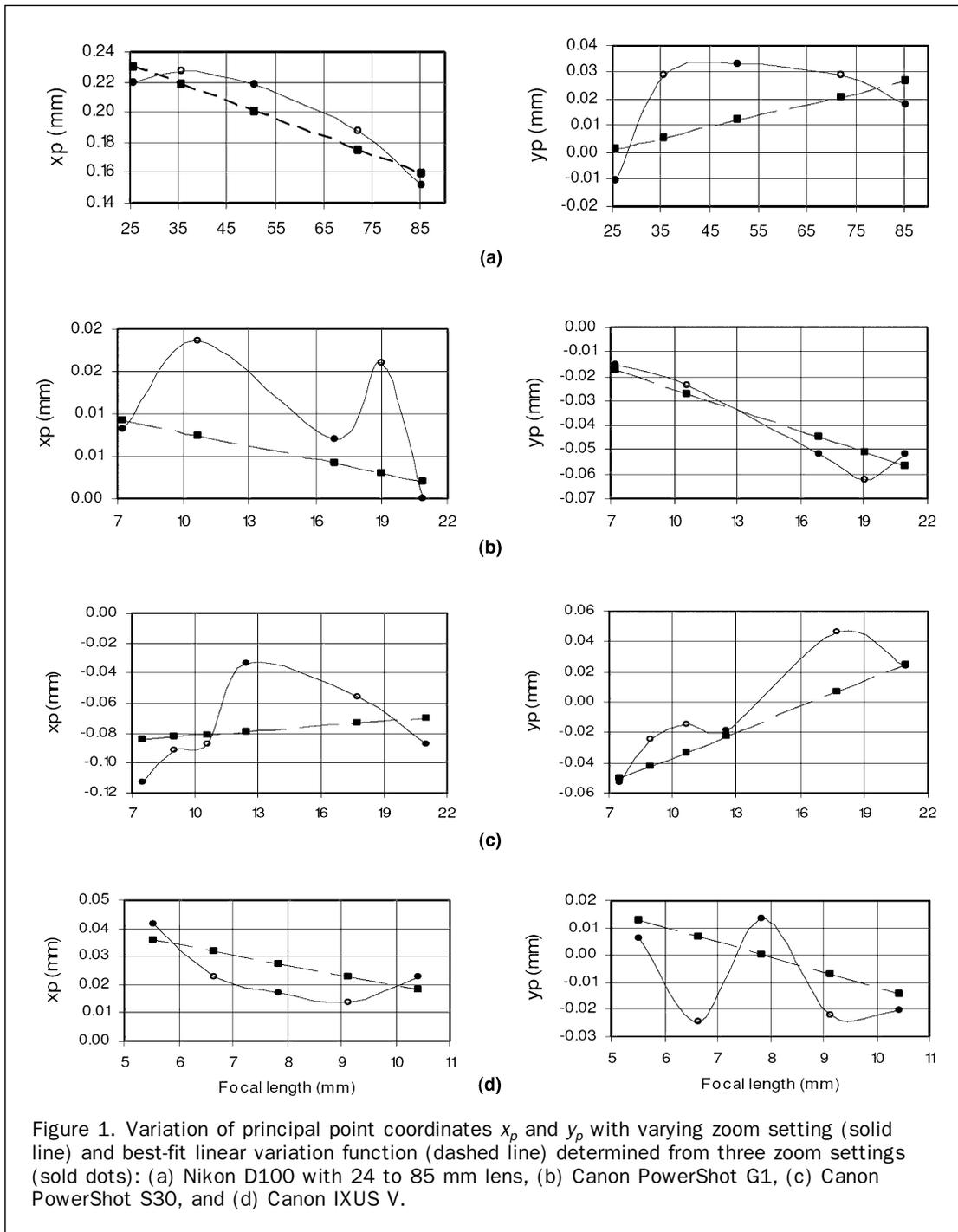
$$K_1^{(c_i)} = d_0 + d_1 c_i^{d_2} \quad (2)$$

where  $c_i$  is the principal distance. In the present case the power of the curve indicated by the coefficient  $d_2$  ranged from -0.2 to -3.1.

As opposed to the systematic behavior of  $K_1$  with changing zoom setting, the behavior of  $K_2$  and  $K_3$  is typically quite erratic, especially at increasing zoom where their contribution to radial distortion is decreasing. It is common in computer vision applications and in many moderate accuracy close-range photogrammetric projects to assume that Gaussian radial distortion is a 3<sup>rd</sup> order perturbation only. Thus,  $K_2$  and  $K_3$  are suppressed. We adopt this approach in the proposed Z-D calibration model, though it must be kept in mind that at shorter focal lengths the radial distortion curve may have significant 5<sup>th</sup> or 7<sup>th</sup> order components.

### Decentering Distortion

Although experimental investigations have indicated that decentering distortion varies with focus and zoom (e.g., Fryer and Brown, 1986; Wiley and Wong, 1995), there have thus far been no reported attempts to model the variation in either the parameters  $P_1$  and  $P_2$  or the decentering distortion profile,  $P(r) = (P_1^2 + P_2^2)^{1/2} r^2$ , as a function of focal length. There are good reasons for this. First, decentering distortion is generally very small, with its significance only becoming apparent in higher accuracy photogrammetric measurement. Second, a significant component of the decentering distortion found in short-pitch zoom lenses can be absorbed within the self-calibration process by the principal point coordinates



$x_p$  and  $y_p$ , especially at longer focal lengths. Shown in Figure 4 are the decentering distortion profiles for the four digital cameras and zoom settings being considering. The variation in  $P(r)$  with zoom shown by the curves, which represent the profile of maximum asymmetric distortion, is well illustrated. It is noteworthy that the decentering distortion profile value is less than one pixel for all but the outer edges of the image format.

Numerous experiments have indicated that for metric application of consumer-grade digital cameras, the omission of decentering distortion parameters from the self-calibrating bundle adjustment has no practical effect on the accuracy of object point determination in a multi-image photogrammetric network. Thus, decentering distortion parameters

will not be considered in the Z-D calibration process being proposed. The user of the technique can always determine the metric impact of this omission, since it is still required to fully self-calibrate the camera at three or more zoom settings. It is a simple matter to ascertain the impact of decentering distortion parameters on photogrammetric triangulation accuracy within these self-calibrations.

### Z-D Calibration Process

Based on both the general behavior of photogrammetric camera calibration parameters for consumer-grade digital cameras, and on the supporting results obtained in the calibrations of the four digital cameras under consideration,

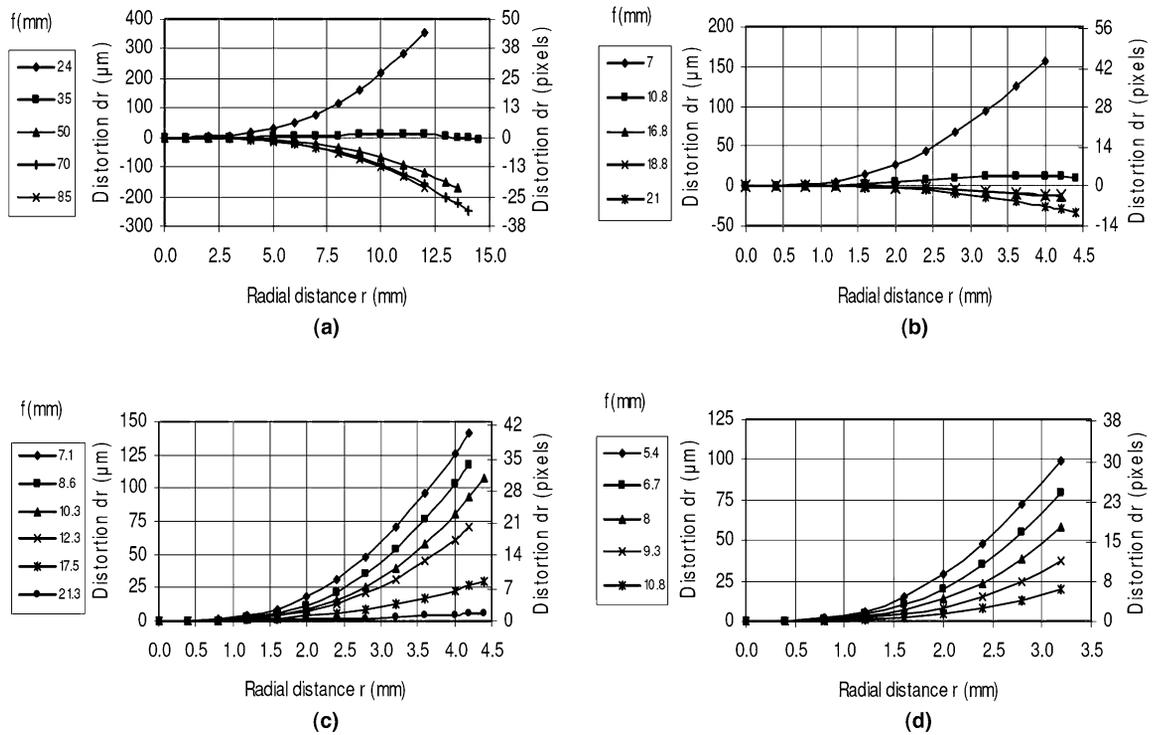


Figure 2. Variation of Gaussian radial distortion with zoom setting: (a) Nikon D100 with 24 to 85 mm lens, (b) Canon PowerShot G1, (c) Canon PowerShot S30, and (d) Canon IXUS V.

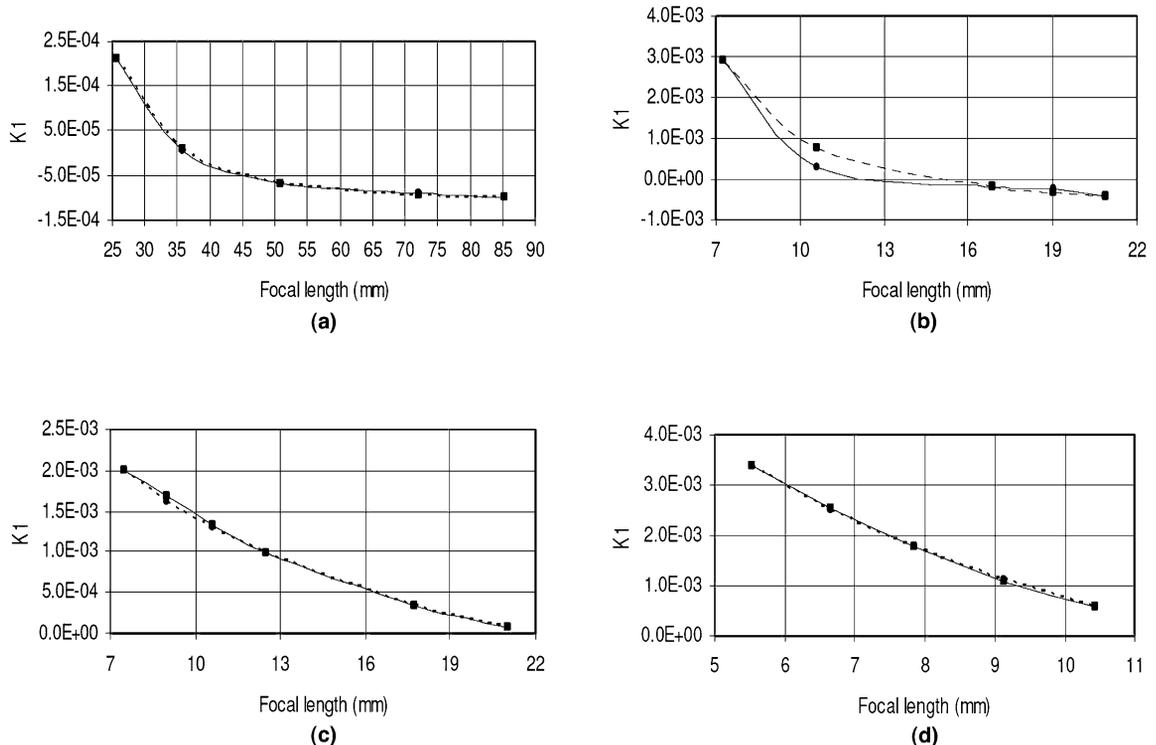


Figure 3. Variation of lens distortion coefficient  $K_1$  with changing zoom setting (solid line) and variation function for  $K_1^{(c1)}$  (dashed line): (a) Nikon D100 with 24 to 85 mm lens, (b) Canon PowerShot G1, (c) Canon PowerShot S30, and (d) Canon IXUS V.

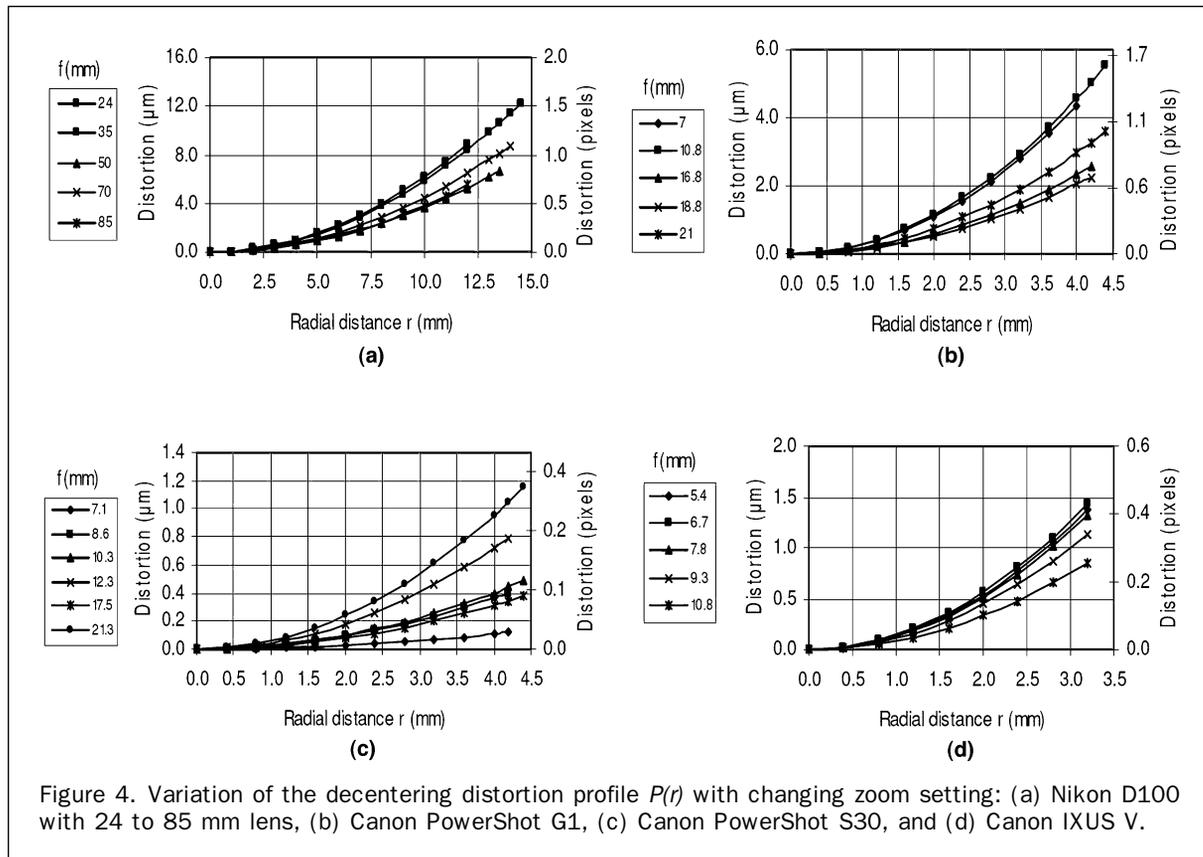


Figure 4. Variation of the decentering distortion profile  $P(r)$  with changing zoom setting: (a) Nikon D100 with 24 to 85 mm lens, (b) Canon PowerShot G1, (c) Canon PowerShot S30, and (d) Canon IXUS V.

we propose a Z-D calibration model where the associated image coordinate correction function is as follows:

$$\begin{aligned} x^{corr} &= x - x_p^{(c_i)} + (x - x_p^{(c_i)})K_1^{(c_i)}r^2 \\ y^{corr} &= y - y_p^{(c_i)} + (y - y_p^{(c_i)})K_1^{(c_i)}r^2. \end{aligned} \quad (3)$$

Here, the radial distance is given as  $r = \left( (x - x_p^{(c_i)})^2 + (y - y_p^{(c_i)})^2 \right)^{0.5}$ ;  $x$  and  $y$  being the measured image coordinates and  $x^{corr}$  and  $y^{corr}$  being the corrected coordinates. The individual Z-D calibration parameters are obtained as follows:

- Principal distance:  $c_i = a_0 + a_1 f_i$  (4)

- Principal point offsets:  $\begin{aligned} x_p^{(c_i)} &= b_0 + b_1 c_i \\ y_p^{(c_i)} &= b_2 + b_3 c_i \end{aligned}$  (5)

- Radial lens distortion coefficient  $K_1^{(c_i)}$ : as per Equation 2.

The linear variation functions in Equations 4 and 5 have coefficients  $a_i$  and  $b_i$ . Two full self-calibrations, nominally at a short and long focal length setting of the zoom lens, are required to determine these coefficients. Self-calibration of the camera at a third zoom setting is necessary for the determination of  $K_1^{(c_i)}$ . However, there is nothing to preclude self-calibration results from additional zoom settings being employed to improve the precision of the modelling.

For many readers the Z-D calibration function, Equation 3, might appear deceptively simple. Yet, it has the potential of freeing the user of digital close-range photogrammetric measurement from the traditional restrictions of recording either all or sufficient sub-sets of images at fixed focus and zoom. So long as the camera records the focal length in the

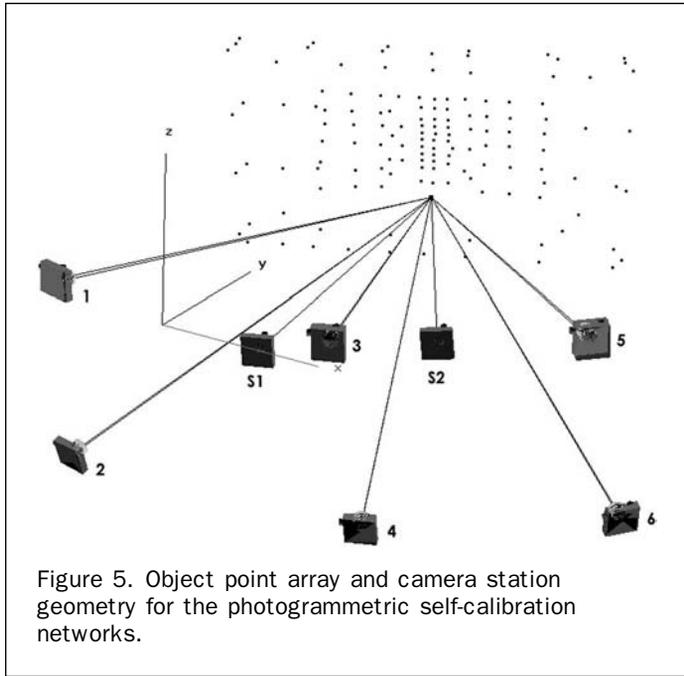
EXIF header for each image, there need no longer be any restriction upon focus or zoom setting.

### Experimental Evaluation

In order to assess the validity of the Z-D calibration approach for consumer-grade digital cameras, a comprehensive series of test measurements at multiple zoom settings was carried out in which the accuracy in 3D object point determination produced by the Z-D calibration approach was evaluated against that produced using a self-calibrating bundle adjustment. A total of eight cameras were tested, though the results of only four are presented here. The outcome of the tests with the remaining consumer-grade cameras was fully consistent with the findings summarized in the next section.

### Test Field

Figure 5 depicts the 140 point test field utilized in the experimental work. This object point array, comprised of retro-reflective targets, covers an area of approximately  $5 \text{ m} \times 3 \text{ m}$ . All object points had previously been measured, again photogrammetrically, to a 3D positional accuracy (RMS 1-sigma) of 0.04 mm. The camera station network shown in the figure is representative of that employed for each camera and zoom setting. A basic convergent network of six stations was used, at which two images per station were recorded with a roll angle difference of  $90^\circ$ . Two additional images were recorded for the purpose of evaluating the Z-D calibration correction approach in two-image stereo networks. As can be appreciated, with the range of focal lengths employed being 5.5 to 85 mm, the full object point array was not imaged in all networks, with the number of object points forming a network varying from all 140 to 55 for the 85 mm



focal length case. As a consequence of the very different imaging scales involved, results of the network adjustments will be presented in scale independent as well as absolute units.

#### Determination of Z-D Calibration Parameters

For three of the four cameras, the D100, G1, and IXUS, photogrammetric networks were recorded at five focal settings throughout the zoom range of the lens, including at the hard limits of zoomed fully out and in. In the case of the S30, six zoom settings were utilized. All photogrammetric measurements were performed automatically using the Australis software system (Fraser and Edmundson, 2000). Every effort was made to ensure optimal image mensuration accuracy, even though the Z-D calibration approach is not really intended for high-accuracy applications. It was desired

to ensure as much as possible that accuracy distinctions between the traditional and Z-D approaches would reflect only the effects of the empirical modelling of the camera calibration parameters.

The procedure for determining the Z-D calibration parameters for each camera was then as follows:

1. A self-calibration was performed for each network, in which the APs of  $c$ ,  $x_p$ ,  $y_p$ ,  $K_1$ ,  $K_2$ ,  $K_3$ , and  $P_1$  and  $P_2$  were employed.
2. A self-calibration was carried out with only  $c$ ,  $x_p$ ,  $y_p$ , and  $K_1$  as APs.
3. The Z-D calibration parameters were determined as per Equations 2 through 5 from the self-calibrations in (2) at three zoom settings only, namely zoomed fully out, mid-zoom, and zoomed fully in.
4. The empirically derived Z-D calibration parameters were then computed for the remaining two zoom settings (remaining three in the case of the PowerShot S30) and standard bundle adjustments were performed for these networks. The focus of the accuracy assessment was upon these surveys since the image correction procedure was using the Z-D approach and fully independent of the corresponding self-calibration results.

## Results

#### Accuracy Attained with the Four Cameras

Shown in Tables 2 through 5 are summaries of results for the four cameras. Part (a) of each table summarizes the outcome of the bundle adjustment with four self-calibration parameters only ( $c$ ,  $x_p$ ,  $y_p$ , and  $K_1$ ) for each zoom setting. Part (b) lists the results obtained in applying Z-D calibration in the networks where the self-calibrated values were not employed in the empirical modelling process. The tables afford an assessment of the impact of Z-D calibration on internal and external accuracy. For internal accuracy, the triangulation misclosures (RMS of image coordinate residuals) can be compared for the Z-D calibration and the corresponding self-calibration, and a measure of the effect on object space coordinate determination of the Z-D calibration approach is provided by the discrepancy between the  $X, Y, Z$  coordinates obtained in each approach, which is provided in Part (b) of each table. With regard to absolute accuracy, the last column of Parts (a) and (b) of the tables lists perhaps the most important accuracy measure, the RMSE against the

TABLE 2. RESULTS OF Z-D CALIBRATION APPLIED TO THE NIKON D100 WITH 24 TO 85 MM LENS

(a) Results of Self-calibrations with the Four APs of  $c$ ,  $x_p$ ,  $y_p$  and  $K_1$ . The Shaded Rows are Zoom Settings not Used for Empirically Determining Z-D Calibration Parameters

Focal Length (mm)	Number of Object Points	RMS of xy Image Coordinate Residuals (pixels)	Mean Std. Error of XYZ (mm)	Diameter of Object (mm)	RMSE Against True XYZ Coordinates in mm (proportional accuracy, 1:xx,000)
24	126	0.11	0.11	5070	0.54 (9)
35	110	0.10	0.07	3920	0.33 (12)
50	83	0.07	0.04	2780	0.24 (12)
70	60	0.07	0.03	2020	0.07 (28)
85	55	0.07	0.02	1490	0.06 (25)

(b) Results of Applying the Z-D Calibration Correction Within the Bundle Adjustment

Focal Length (mm)	RMS of xy Image Coordinate Residuals (pixels)	Mean Std. Error of XYZ (mm)	RMSE of XYZ Coords. Against Self-cal with Four APs, (mm)	RMSE Against True XYZ Coordinates in mm (proportional accuracy, 1:xx,000)
35	0.10	0.07	0.08	0.33 (12)
70	0.07	0.02	0.02	0.07 (28)

TABLE 3. RESULTS OF Z-D CALIBRATION APPLIED TO THE CANON POWERSHOT G1

(a) Results of Self-calibrations with the Four APs of  $c$ ,  $x_p$ ,  $y_p$  and  $K_1$ . The Shaded Rows are Zoom Settings not Used for Empirically Determining Z-D Calibration Parameters

Focal Length (mm)	Number of Object Points	RMS of xy Image Coordinate Residuals (pixels)	Mean Std. Error of XYZ (mm)	Diameter of Object (mm)	RMSE Against True XYZ Coordinates in mm (proportional accuracy, 1:xx,000)
7	135	0.15	0.22	5070	0.52 (10)
10.8	114	0.12	0.13	4480	0.37 (12)
16.8	81	0.06	0.03	2770	0.12 (23)
18.8	74	0.05	0.03	2450	0.12 (23)
21	65	0.06	0.03	2450	0.09 (27)

(b) Results of Applying the Z-D Calibration Correction Within the Bundle Adjustment

Focal Length (mm)	RMS of xy Image Coordinate Residuals (pixels)	Mean Std. Error of XYZ (mm)	RMSE of XYZ Coords. Against Self-cal with Four APs, (mm)	RMSE Against True XYZ Coordinates in mm (proportional accuracy, 1:xx,000)
10.8	0.22	0.26	0.64	0.59 (8)
18.8	0.08	0.05	0.13	0.14 (18)

true coordinate values, along with the equivalent proportional accuracy.

It is useful to highlight aspects of the results prior to summarizing the outcome of the Z-D calibration model for each camera. First, considering that with the possible exception of the D100 with 24 to 85 mm lens, the cameras are demonstrably in the inexpensive consumer-grade category, they produce relatively high 3D point determination accuracy. Moreover, the accuracy, as should be anticipated, is poorest at the shortest focal length and generally best at the largest zoom setting. For the smallest zoom, widest field of view setting, proportional accuracies in 3D object point determination range from 1:9,000 to 1:17,000, whereas at the longest focal length they range from 1:25,000 to 1:40,000. Surprisingly, the most consistently accurate camera for all zoom settings was that with the lowest resolution, namely the 2-megapixel IXUS V.

Second, the relatively high object point determination accuracy is largely a function of the high precision of image coordinate measurements. The RMS value of image coordinate residuals in the self-calibration adjustments ranged from a maximum of 0.15 pixels to a minimum of 0.05 pixels, again with the precision improving with increasing zoom. Readers should recall that manual image coordinate measurement would be likely to yield residuals of an order of magnitude larger than this, i.e., 0.3 to 1.5 pixels, which would commensurately bring the 3D object point accuracy down to a range of about 1:1,000 to 1:4,000 for the same network geometry.

Third, it is interesting to compare the triangulation misclosure values from the full (8 APs) and partial (4 APs) self-calibrations (i.e., from Steps 1 and 2 above). In regard to image coordinate misclosure, RMS values within 0.02 pixel were obtained for all cameras for all but the shortest focal

TABLE 4. RESULTS OF Z-D CALIBRATION APPLIED TO THE CANON IXUS V

(a) Results of Self-calibrations with the Four APs of  $c$ ,  $x_p$ ,  $y_p$  and  $K_1$ . The Shaded Rows are Zoom Settings not Used for Empirically Determining Z-D Calibration Parameters

Focal Length (mm)	Number of Object Points	RMS of xy Image Coordinate Residuals (pixels)	Mean Std. Error of XYZ (mm)	Diameter of Object (mm)	RMSE Against True XYZ Coordinates in mm (proportional accuracy, 1:xx,000)
5.4	139	0.08	0.15	5070	0.30 (17)
6.7	134	0.07	0.12	4980	0.27 (18)
8.0	127	0.07	0.10	4570	0.22 (21)
9.3	112	0.06	0.08	3620	0.15 (24)
10.8	110	0.05	0.06	3750	0.12 (31)

(b) Results of Applying the Z-D Calibration Correction Within the Bundle Adjustment

Focal Length (mm)	RMS of xy Image Coordinate Residuals (pixels)	Mean Std. Error of XYZ (mm)	RMSE of XYZ Coords. Against Self-cal with Four APs, (mm)	RMSE Against True XYZ Coordinates in mm (proportional accuracy, 1:xx,000)
6.7	0.09	0.15	0.30	0.27 (18)
9.3	0.06	0.08	0.08	0.13 (28)

TABLE 5. RESULTS OF Z-D CALIBRATION APPLIED TO THE CANON POWERSHOT S30

(a) Results of Self-calibrations with the Four APs of  $c$ ,  $x_p$ ,  $y_p$  and  $K_1$ . The Shaded Rows are Zoom Settings not Used for Empirically Determining Z-D Calibration Parameters

Focal Length (mm)	Number of Object Points	RMS of $xy$ Image Coordinate Residuals (pixels)	Mean Std. Error of XYZ (mm)	Diameter of Object (mm)	RMSE Against <i>True</i> XYZ Coordinates in mm (proportional accuracy, 1:xx,000)
7.1	138	0.11	0.13	5070	0.41 (12)
8.6	131	0.07	0.08	5070	0.18 (28)
10.3	115	0.07	0.06	4430	0.12 (37)
12.3	100	0.06	0.04	3920	0.08 (49)
17.5	74	0.05	0.03	2450	0.08 (31)
21.3	64	0.05	0.02	2020	0.05 (40)

(b) Results of Applying the Z-D Calibration Correction Within the Bundle Adjustment

Focal Length (mm)	RMS of $xy$ Image Coordinate Residuals (pixels)	Mean Std. Error of XYZ (mm)	RMSE of XYZ Coordinates Against Self-cal with Four APs, (mm)	RMSE Against <i>True</i> XYZ Coordinates in mm (proportional accuracy, 1:xx,000)
8.6	0.09	0.10	0.24	0.33 (15)
10.3	0.08	0.06	0.10	0.14 (32)
17.5	0.06	0.03	0.05	0.07 (35)

length. For the smallest zoom setting the discrepancies ranged from 0.01 to 0.06 pixels, the higher value for the four AP case no doubt reflecting the influence of omitting the  $K_2$  and  $K_3$  radial distortion terms. As regards absolute accuracy, the difference in the full versus partial self-calibrations reached a maximum at the shortest focal setting, the discrepancy being equivalent to 1:20,000 of the object field diameter. For all other zoom settings the accuracy difference was 1:40,000 or less, which is not regarded as significant.

Finally, it is encouraging to observe that for the mid-range and longer zoom focal lengths, the discrepancy between the object space coordinates obtained in the Z-D calibration approach and those from the self calibration is small. The difference is largest for the short focal lengths.

Turning now to the individual cameras, the first noteworthy feature of the D100 results shown in Table 2 is the range of absolute accuracies in  $X,Y,Z$  coordinates achieved in the self-calibrations, namely 1:9,000 to 1:28,000. The relatively poorer results for the shortest focal length are almost certainly a consequence of un-modelled decentering distortion and higher-order radial distortion effects, both of which were significant for the 24 to 85 mm zoom lens. The accuracy figures for the longer focal lengths may understate the true situation since these networks display equal or better object point precision than the assumed *true* coordinates. In the two cases where the empirically modelled Z-D parameters were applied, the accuracy attained matched that from the corresponding self-calibration adjustments.

The PowerShot G1 also displays significant fifth-order radial distortion at the shortest zoom focal length. Thus, we see in Table 3b a significant increase in the RMS value of image coordinate residuals for the 10.8 mm focal length as compared to the corresponding self-calibration, which by itself is higher than expected due to the omission of the  $K_2$  and  $K_3$  coefficients from the AP model. Also, the absolute accuracy achieved when applying the Z-D calibration parameters falls to 1:8,000, as opposed to 1:12,000 in Table 3a. A drop in accuracy is also seen at the 18.8 mm focal length, but here it is more modest.

As previously mentioned, the IXUS camera gave the most consistent results when assessed in terms of achievable accuracy through the Z-D calibration approach. As can be

seen from Table 4, there was no difference in photogrammetric triangulation accuracy between the Z-D calibration and self-calibration solutions. Moreover, this 2-megapixel camera with its  $2 \times$  optical zoom lens produced absolute accuracies of 1:17,000 or better for all zoom settings.

With the PowerShot S30 we again see a close equivalence in attainable accuracy between the Z-D calibration and self-calibration approaches. As shown in Table 5, although the RMS discrepancy in object point coordinates is largest at the 8.6 mm zoom setting, an accuracy of 1:15,000 is still achieved. At the mid-zoom and zoomed-in settings, this figure improves to better than 1:30,000.

#### Mixed Focus/Zoom Settings

One of the main aims of Z-D calibration is to facilitate photogrammetric networks in which images can be recorded at any focus/zoom setting. As a final test of the Z-D calibration approach, the technique was applied to three network configurations of PowerShot S30 images, these being as follows:

1. A six-station network with two images each from zoom settings of 8.6, 10.3, and 17.5 mm. The stations occupied (approximately) the positions 1, 2, S1, S2, 5, and 6 in Figure 5.
2. A four-station geometry with two images each from 8.6 and 10.3 mm focal lengths, the stations corresponding to positions 1, 2, 5, and 6 in Figure 5.
3. A two-station, nominally stereo configuration at a zoom focal length of 17.5 mm, with the positions being at S1 and S2 in Figure 5.

Shown in Table 6 for all three networks are the results of the bundle adjustments which utilized the empirically modelled Z-D calibration parameters. When it is recalled that the images from the three chosen zoom settings played no part in the determination of Z-D calibration parameters, the results are quite impressive. In all cases the absolute accuracy attained, while being less than that listed for the 12-station networks in Table 5, exceeds 1:15,000, even for the stereo configuration. Moreover, the triangulation misclosure values correspond well to those from the self-calibration adjustments, being better than 0.1 pixel in all three networks. These

TABLE 6. RESULTS OF APPLYING EMPIRICAL CALIBRATION MODEL WITH BUNDLE ADJUSTMENT TO DIFFERENT FOCAL SETTING COMBINATIONS FOR THE POWERSHOT S30

Focal Lengths (mm)	RMS of xy Residuals (pixels)	Mean std. Error of XYZ (mm)	RMSE Against True XYZ Coordinates in mm (proportional accuracy, 1:xx,000)
2@each = 6	0.07	0.26	0.32 (16)
2@8.6 + 2@10.3 = 4	0.07	0.27	0.33 (15)
2@17.5	0.02	0.07	0.11 (17)

results highlight the potential of the Z-D calibration approach for networks incorporating images from different zoom settings.

## Conclusions

The results of this investigation show that the Z-D calibration approach is viable for consumer-grade digital cameras, and indeed can produce 3D object point positioning accuracy well beyond that usually associated with such cameras, and certainly beyond the demands of many users of close-range photogrammetry. The Z-D calibration process is quite straightforward to implement, especially if one has the capability of fully automatic camera calibration, as exemplified by the “iWitness” system (Fraser and Hanley, 2004). “iWitness” employs color-coded targets and automatic camera identification to provide a virtually scale independent on-site camera self-calibration in a matter of minutes, with no prior knowledge of the camera being required.

Recent studies (Laebe and Foerstner, 2004; Habib *et al.*, 2004), supported by the authors’ experience, indicate that the calibration parameters of consumer-grade digital cameras, especially the all-important lens distortion, are relatively stable over time and thus the process of Z-D parameter determination is likely to be infrequently required. When in doubt, an automatic self-calibration can easily indicate the validity of the current Z-D calibration parameters.

The real benefit of the Z-D calibration approach is practical; once the image coordinate correction parameters are established, and there are supporting data processing facilities, the user of close-range photogrammetry is freed from the restrictions of capturing images at fixed zoom/focus settings. This can greatly enhance the flexibility of moderate accuracy close-range photogrammetric measurement and potentially further broaden the applications base for the technology.

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