THOUGHTS ON A STANDARD ALGORITHM FOR CAMERA CALIBRATION

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Abstract

Starting from concerns related to the calibration of aerial cameras, several aspects of a standardization of camera calibration procedures are discussed. These include a general review of standardization activities and a possible role of *ISPRS* in such activities, a definition of the term *camera calibration* and summaries of a mathematical model and an algorithm for camera calibration. The mathematical model includes special considerations for cameras with adjustable focus.

Introduction

Aerial cameras have been routinely calibrated for several decades in a larger number of countries using procedures developed to meet the needs of a central mapping agency. The desirability of a more standardized approach to camera calibration in regard to calibration procedures and calibration reports has in recent years become increasingly important, in particular in view of the camera calibration needs of countries where a calibration facility is not available. Such countries rely on calibration services offered elsewhere and are anxious to obtain comparable data about a camera from repeated calibrations even if these are carried out by different laboratories.

This paper looks at standards and how they are produced, reviews definitions for *camera calibration*, proposes a standard image coordinate system, introduces a general mathematical model for photogrammetry, suggests definitions for the rotationally-symmetrical and decentring components of the lens distortion for aerial cameras, summarizes suggestions for extentions to the lens distortion definitions for close-range applications and introduces an algorithm for the calibration of aerial cameras.

Standards

A standard is defined in [6] as

> " ... 3a: something that is established by authority, custom, or general consent as a model or example to be followed: CRITERION, TEST b: a definite level or degree of quality that is proper and adequate for a specific purpose 4: something that is set up and established by authority as a rule for the measure of quantity, weight, extent, value, or quality ..."

Standards are being developed by national and international, civilian and military agencies. The development of standards falls into two categories: the standardization of industrial parts and the standardization of testing techniques. The first category aims at achieving compatibility between parts and instruments made by different manufacturers. Second category standards aid in making possible meaningful comparisons between different products; an example for such a standard are photographic lens test specifications.

In particular national civilian standards are the result of voluntary activities benefitting from the input of possibly several different national technical societies which in turn may be supported by manufacturers and other interested groups, and international civilian standards from the input of national standards organizations and possibly also international professional societies which may enter into a formal liaison agreement. Most standards result from compromises by all parties participating in their development. These compromises may sometimes be difficult to accept, particularly when prior standards exist, and when people and countries have to change the way in which they have been doing things. It is important that the fundamental integrity of a standard is maintained in spite of such compromises to prevent standards from becoming meaningless pieces of paper.

International military standards, such as those developed by the North Atlantic Treaty Organization (*NATO*), are also the result of compromises achieved by the member countries; the *NATO* standards (known as Standardization Agreements or *STANAG*'s) are developed by so-called Working Parties with the objectives of enhancing of the operational effectiveness of the *NATO* forces, facilitating cross operations, enhancing the effectiveness of cross servicing, facilitating the cross training of personel and facilitating the exchange of information or equipment. While these objectives do not fully agree with those for civilian standardization efforts, it is nonetheless desirable that civilian and military standards agree to the extent possible. Separate government standards, be they civilian or military, should be avoided if possible since, for example in the U.S.A. and Canada, federal agencies are invited to participate in the development of both national and international (for example *ISO* or *NATO*) standards. To do otherwise is often a waste and a source of confusion and antagonism. It often leads to standards which are incompatible.

A short history and a short review of the activities of the International Organization for Standardization (*ISO*) were given in [21]. The *ISO* activities of interest in view of camera calibration were first pursued by the Technical Committee (*TC*) 42 (Photography) and thereafter, after the formation of the *TC* 172 (Optics and Optical Instruments) in 1979, gradually transfered to that *TC* to eliminate a duplication of efforts. *TC* 172 has nine Study Groups (*SG*) dealing with (1) fundamental standards, (2) materials for optical processing, (3) optical materials and components, (4) telescopes, (5) microscopes, (6) geodetic instruments, (7) ophthalmic, endoscopicand metrological instruments and test methods, (8) ophthalmic optics and (9) electro-optical systems. Recent activities of Working Group 1 of *SG* 1 included the development of standards for definitions and principles of measurement for optical transfer functions, and the drafting of proposed standards for a test method for the veiling glare of photographic objectives and for the determination of the image distortion of photographic lenses.

Standards for optical components and systems and for the application of such systems have also been in development by various other standard organizations. While these standards are not directly applicable to photogrammetric camera calibration, they could well be used as a base for the development of special recommanded procedures or, for example, *ISO* application standards as this is at present considered for OTF determination. The *ISPRS* statutes ([1]) were also reviewed in [21] and it was concluded that they do not explicitely cover any involvement in the development of standards, but that such an activity is certainly permitted under the following paragraph:

> " The Society may do all other things incidental or conductive to the Society's aims, provided such acts do not contravene the Statutes or By-laws of the Society, or the laws of the country in which they are done, or the general principles to which the Society subscribes."

The *ISPRS* "Recommended Procedures for Calibrating Photogrammetric Cameras and for Related Optical Tests" ([4]) are clearly not a standard although they too are a compromise achieved after sometimes intense discussions lasting over two decades. Their revision could possibly based on available *ISO* standards and thus take, to a certain extent, advantage of the periodic review process which all *ISO* standards undergo.

What is Camera Calibration?

The meaning of *to calibrate* according to [6] which is most appropriate within the present context, is

> " 1 ... c: to standardize (as a measuring instrument) by determining the deviation from standard especially as to ascertain the proper correction factors ..."

From a photogrammetric point of view it can be stated ([11], p. 92):

> " Basically, the purpose of camera calibration is to be able to reconstruct the precise geometry of the bundle of rays that entered the camera at the instant of exposure from the two-dimensaional measurement of points on the resulting photograph."

Camera calibration may mean different things to different photogrammetrists if these were educated in different traditions. For example, the continental European understanding of the term typically includes the determination of "... the elements of the interior orientation ... [which include] the location of the principal point with respect to the fiducial marks, the camera constant and the lens distortion with respect to that constant" ([16], pp. 55+56). A more recent treatment ([7], p. A 3.1.1-6) lists the elements of the interior orientation which are to be reported in a calibration certificate as

- the coordinates of and distances between the fiducial marks,
- the coordinates (in the same image coordinate system) of the principal point of autocollimation, the point of best symmetry and the fiducial centre,
- the camera constant,
- the average radial lens distortion,
- the date of calibration and
- (unspecified) information about the image sharpness.

The *ISPRS* "Recommended Procedures for Calibrating Photogrammetric Cameras and for Related Optical Tests" ([4]) concur essentially with this usage of the term *camera calibration*: the document originally consisted of the sections (1) *resolving power*, (2) **calibration**, (3) *veiling glare*, (4) *image illumination* and (5) *deviation of filters and camera port* *glasses*. A sixth section entitled *modulation transfer function and optical transfer function of photogrammetric lenses and guidelines for measuring MTF of camera systems* (part of [12]) was later added.

In North America, the term camera calibration usually also includes the determination of some measure of image quality. According to ([11], p. 91), a calibration results in "metrical characteristics" which include

- the focal length of the camera lens,
- the radial and tangential distortions of the lens,
- the resolving power of the lens,
- the position of the principal point with respect to the fiducial marks,
- the flatness of the focal plane and
- the relative positions of, or the distances between, the fiducial marks (this may include the determination of the position of réseau marks in respect to the fiducial marks).

The latest edition of the *ASPRS* Manual of Photogrammetry describes camera calibration in chapter XIX "Definitions of Terms and Symbols Used in Photogrammetry" ([17], p. 1003) as follows:

> " The determination of the calibrated focal length, the location of the principal point with respect to the fiducial marks, the point of symmetry, the resolution of the lens, the degree of flatness of the focal plane, and the lens distortion effective in the focal plane of the camera and referred to the particular calibrated focal length... The setting of the fiducial marks and the positioning of the lens are ordinarily considered as adjustments, although they are sometimes performed during the calibration process. Unless a camera is specifically referred to, distortion and other optical characteristics of the lens are determined in a focal plane located at the equivalent focal length and the process is termed lens calibration. In close-range photogrammetry, calibration may be performed directly or indirectly on the camera, the individual photograph (particularly in the case of non-metric cameras) or on the total system. Camera calibration may be done in one of three forms: laboratory, on-the-job, and self-calibration. Laboratory calibration of a camera is performed separately from the photography phase and is undertaken with goniometers or test areas [arrays?] of various sophistication. On-the-job calibration of a camera or a photograph utilizes object photography and object-space control points. Self-calibration of a camera or a photograph utilizes object photography and well-defined object points. Calibration of a system may be accomplished by including parameters such as affinity and non-perpendicularity of comparator axes."

The before mentioned meaning of *to calibrate* suggests that the term *camera calibration* could be used either limited to the determination of the (interior orientation) parameters defining the geometric-optical performance of a camera or with the extended meaning given, for example, in the *ASPRS* Manual of Photogrammetry. It is therefore important that attempts be made within *ISPRS* to arrive at a definition of the term universally acceptable. This is also important in view of the fact that other disciplines may interpret the term altogether different.

Papers dealing with robot vision, for example, separate between *modeling the cameras* and *calibrating the cameras*; modeling the cameras may range "... from the simple pin-hole model to such models which include the addition of the depth of field due to finite aperture camera, spatial aberration, geometric distortions, spectral aberration, or other non-ideal characteristics of physical lenses ..." ([10], p. 277, with reference to [13]). On the other hand, the calibration of a camera is considered to consist of the determination of the parameters of the exterior orientation, although attempts are made to model during this process deviations from the "pin-hole" model in the image plane using functional relationships (for example in [9]). Since non-photographic cameras will in future find increasing use in photogrammetric work which may include the development of algorithms for robot vision, it is important to be aware of and possibly modify this usage of the term *camera calibration*.

A Standard Image Coordinate System

A plane image surface is assumed for the following discussion, and positional changes resulting from any deviations of the surface from a plane will be considered as part of the image errors. It is desirable to define a unique image coordinate system for the image plane if asymmetrical components are part of the parameter group defining the interior orientation of a camera. A workshop held by WG I/2 in Stockholm during June 1985 agreed for aerial film cameras that

> " ... the x-axis should coincide with the film transport direction and be accurately defined by two fiducial marks to be determined later ... [and that] it should point towards the next photograph on the film. The y-axis should be located at 90 degrees in counterclockwise direction for an emulsion-up positive image. The centre of the coordinate system should be located at the point of intersection of the four primary fiducial marks as defined by the manufacturer (e.g. corner marks in Wild cameras, side middle marks in Zeiss Oberkochen cameras). Suggestions to move one of the fiducial marks on purpose to introduce asymmetry allowing the unique determination of the position of an aerial photograph on the film were mentioned, but the approach

was not considered to be practical. Rather, one of the primary fiducial marks might be marked or shaped such that it can serve the same purpose. It was further discussed that a unique numbering scheme for fiducial marks would be desirable; the camera manufacturers were to be approached in this regard."

The foregoing statements could be modified to cover other camera types as long as the basic intent of providing a unique definition of an image coordinate system is retained. It is expected that clarification on some as yet unresolved matters can be achieved during the forthcoming Stuttgart symposium of Commission I and that a final agreement may be reached during the forthcoming Kyoto congress.

The origin of a photogrammetric bundle deviates from that of an image in two ways; it may not be identical within the image plane and it is located at a certain distance away from that plane. Deviations within the image plane result from differences between the chosen mathematical projection and the physical reality. The origin of the coordinate system can be placed into a number of different image-centre reference points located near the centre of the image area such as fiducial centre(s) (as suggested in the preceding citation), a point of best symmetry (of the radial lens distortion component), the principal point of autocollimation or, if a réseau is present, the centre point of the réseau, or it may be chosen outside of the image area if only positive coordinate values are desired. All resired image-centre reference points should then be defined in the same coordinate system. The projection centre required to reconstruct the imaging bundle of rays may be located such that a perpendicular dropped from it to the image plane intersects any one of the different image-centre reference points. The projection centre is distant from the image plane by a *focal length* such as the "equivalent focal length", the "calibrated focal length" or the "camera constant"; the latter two are photogrammetric constants chosen to meet a certain optimization requirement for the rotationally-symmetrical lens distortion. It is obvious that an agreement on a definition and use of the different image-centre reference points and focal lengths and on the direction of the focal-length vector is desirable; it is necessary as a prerequisite for a standardization of the reporting of camera calibration results.

A Mathematical Model

Each measuring procedure requires the formulation of an appropriate mathematical model which permits the derivation of functional relationships between available measurements and parameters desired as the end result of the procedure. Any photogrammetric procedure can be considered as a three-dimensional measuring procedure based on the use of unoriented bundles of spatial directions. Imaging rays originating at object points pass through a lens and then intersect a light-sensitive surface which is capable of recording the point of intersection. The measurements extracted from the recorded image are subject to image-modifying effects of both the lens and the recording medium. The lens may change the direction of the imaging ray, and the recording medium may cause a shift of a recorded point as a result of the interaction between internal structure and incoming light intensity. In addition, the geometry of the recorded image may be altered after the recording; for example, an image on film will be subject to film deformation.

The mathematical model of any mensuration process can in a general way be formulated as

$$F(\mathbf{y}) = 0 \tag{1}$$

([15]) where the vector $\mathbf{y}$ contains both the measurements $\mathbf{l}$ and the parameters or unknowns $\mathbf{u}$

$$F(\mathbf{y}) = F(\mathbf{l}, \mathbf{u}) = 0. \tag{2}$$

Photogrammetrists have traditionally prefered to use the "pin-hole" model and considered deviations from collinearity of object point, centre of projection and image point as errors of the imaging process. Since manufacturers of photogrammetric cameras have made great efforts and progress in the development of lenses approaching this ideal, it is prudent to maintain this model, in particular, since it can be assumed for the majority of applications that all imaging rays entering the lens pass through a single point, the centre of projection. It is then obvious that a mathematical model for the solution of a photogrammetric problem be based on individual imaging rays. Hence, the information content of a single bundle consisting of i rays can be written as

$$I_i = \sum_{i=1}^{n} F(\mathbf{y})_i = 0,$$

and that of a general photogrammetric problem involving j bundles as

$$I_j = \sum_{j=1}^{m} \sum_{i=1}^{n} F(\mathbf{y})_{ij} = 0.$$

Equation (1) can for an imaging system using a single projection centre be written as

$$F(\mathbf{y}) = \mathbf{x} - \lambda \cdot \mathbf{M} \cdot \mathbf{z} = 0$$

where $\mathbf{x}$ is a vector in the image space. λ a scale factor, $\mathbf{M}$ an othogonal matrix and $\mathbf{z}$ a vector in the object space. or as

$$F(\mathbf{y}) = \frac{1}{\lambda} \cdot \mathbf{M}^T \cdot \mathbf{x} - \mathbf{z} = 0 \tag{3}$$

with

$$\mathbf{M}^T = \mathbf{M}_X^T \cdot \mathbf{M}_Y^T \cdot \mathbf{M}_Z^T = \mathbf{R}_X \cdot \mathbf{R}_Y \cdot \mathbf{R}_Z \cdot \mathbf{Q}.$$

The 3x3 matrix $\mathbf{Q}$ is a pre-orientation matrix containing six zero elements and three elements with the value 1 or -1 such that each row and each column contains only one non-zero element. The use of $\mathbf{Q}$ permits the use of any combination of an Cartesian image coordinate system with any object coordinate system and limits the absolute value of any of the non-diagonal elements in the matrices $\mathbf{R}$ to about 0.7; it also obviates the need to have the focal length vector used in the evaluation of vertical aerial photography point towards rather than away from the projection centre.

The solution of the problem should be formulated using mathematical terms as simple as possible but including all parameters which may contribute to the simulation of the photogrammetric imaging process ([15]). Typically, the original relation (2) is linearized using only the linear terms of a Taylor's series:

$$F(\mathbf{l}^0, \mathbf{u}^0) + \left[\frac{\partial F(\mathbf{y})}{\partial \mathbf{l}}\right]_{l^0} \cdot \Delta\mathbf{l} + \left[\frac{\partial F(\mathbf{y})}{\partial \mathbf{u}}\right]_{u^0} \cdot \Delta\mathbf{u} = 0$$

with

$$\mathbf{l} = \mathbf{l}^0 + \Delta\mathbf{l} \quad \text{and} \quad \mathbf{u} = \mathbf{u}^0 + \Delta\mathbf{u}.$$

The measurements and unknowns making up the vector $\mathbf{y}$ in the preceding equations can be grouped as follows:

$$\mathbf{y}^T \doteq (\mathbf{c}_i \; \mathbf{c}_o \; \mathbf{o}_i \; \mathbf{o}_e \lambda)^T$$

where vector $\mathbf{c}_i$ represents the image coordinates which are part of the vector $\mathbf{x}$ in equation (3), vector $\mathbf{c}_o$ the object coordinates (part of $\mathbf{z}$), vector $\mathbf{o}_i$ the elements of the interior orientation (part of $\mathbf{x}$), vector $\mathbf{o}_e$ those of the exterior orientation (partly part of $\mathbf{M}$ and partly part of $\mathbf{z}$) and λ a scale factor. Vector $\mathbf{o}_i$ consists of the coordinates of the image-centre reference point(s), the focal length(s) and the parameters defining the lens distortion.

Lens Distortion for Fixed-Focus Cameras

The image space vector $\mathbf{x}$ in (3) can be rewritten as

$$\mathbf{x} = \mathbf{R} \cdot \mathbf{D} \cdot \mathbf{E} \cdot \mathbf{x}' \tag{4}$$

where matrix $\mathbf{R}$ represents the contribution of the rotationally-symmetrical lens distortion, matrix $\mathbf{D}$ that of the decentring distortion, matrix $\mathbf{E}$ that of the effects of various sources of image deformation and of errors of the measuring instrument, and vector $\mathbf{x}'$ the error-free image-space vector. Matrix $\mathbf{E}$ is here assumed to be an identity matrix, since image deformation and errors of the measuring instrument are outside the scope of the present discussion; in this case vector $\mathbf{x}'$ consists of the two image coordinates and the focal length. It is further

$$\mathbf{R} = \begin{pmatrix} 1 + dr/r & 0 \\ 0 & 1 + dr/r \end{pmatrix} \tag{5}$$

with dr/r expressed by a polynomial, for example,

$$dr/r = s_0 + s_1 r^2 + s_2 r^4 + s_3 r^6 + \ldots, \tag{6}$$

and

$$\mathbf{D} = \begin{pmatrix} 1 + dx & 0 \\ 0 & 1 + dy \end{pmatrix} \tag{7}$$

with dx and dy defined using the Conrady-Brown model as

$$\begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} -\sin\phi & \cos\phi \\ \cos\phi & \sin\phi \end{pmatrix} \begin{pmatrix} dt \\ dr \end{pmatrix} = \frac{1}{r} \begin{pmatrix} -y & x \\ x & y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & m \end{pmatrix} \begin{pmatrix} \cos\phi_0 & \sin\phi_0 \\ -\sin\phi_0 & \cos\phi_0 \end{pmatrix} \frac{1}{r} \begin{pmatrix} x \\ y \end{pmatrix} P \tag{8}$$

where P is again a polynomial, for example,

$$P = d_0 + d_1 r^2 + d_2 r^4 + \ldots \tag{9}$$

and ϕ_0 the orientation angle for the decentring profile. It should be noted that the radial distances r in equations (6), (8) and (9) are not necessarily identical: the rotationally-symmetrical lens distortion is usually determined with respect to a point of best symmetry (*PBS*) while the decentring distortion is often (in laboratory calibration procedures) determined with respect to the principal point of autocollimation (*PPA*). The use of the *PPA* requires an instrumental set-up such that a camera can only be rotated around its axis which, by definition, is perpendicular to the image plane, while the use of the *PBS* requires that a camera can rotate freely in space. Decentring distortion values are also determined using other image-centre reference points than the *PPA*, and the location of the *PBS* may be dependent upon the selection of points used for its determination.

Equation (8) for the decentring lens distortion can also be written as ([5], where $m = 3$ is used)

$$dt = \cos(\phi - \phi_0) \cdot P \quad \text{and} \quad dr = m \cdot \sin(\phi - \phi_0) \cdot P$$

or, if ϕ_0 is replaced by $\phi_0 = \phi_0' - 90^\circ$, as

$$dt = \sin(\phi - \phi_0') \cdot P \quad \text{and} \quad dr = m \cdot \cos(\phi - \phi_0') \cdot P.$$

The tangential lens distortion component is of maximum size for $\phi = \phi_0$ and disappears for $\phi = \phi_0'$. Investigation of the lens distortion of *Jena UMK A 10/1318* terrestrial cameras showed evidence of the existence of affine lens distortion, possibly caused by toroidal deformations of lens elements ([19]). If an average focal length is determined first, than two extreme magnification factors, $(1+v)$ and $(1-v)$, are observed. The affine lens distortion can be formulated in a fashion similar to the last equation for the decentring distortion as

$$dt = \sin 2(\Phi - \Phi_0) \cdot v \cdot r \quad \text{and} \quad dr = \cos 2(\Phi - \Phi_0) \cdot v \cdot r \tag{10}$$

where Φ_0 is the orientation angle for the radius where $(1+v)$ occurs.

The lens distortion polynomials given in equations (6) and (9) can be modified by elimination of some of the parameters s_i or d_i to adopt the model to practically all the different mathematical models presently in use for the calibration of aerial cameras. In particular, often only the term $d_1 r^2$ of the decentring polynomial is used, for example in [5] and [19]. Although polynomials have proven to be essential for a proper modelling of the lens distortion, they are undesirable from a mathematical point of view because of the high correlation between the different terms. It is therefore necessary to agree not only on the formulations for the lens distortion components but also on the procedure used to solve for these parameters.

Lens Distortion for Cameras with Adjustable Focus

The calibration of cameras with adjustable focus differs from the calibration of fixed-focus cameras in two aspects: (1) the lens distortion changes as a function of object distance, and (2) the centre of the entrance pupil and the frontal cardinal point can no longer be considered identical. Other errors, for example the film flattening in non-metric cameras, would be included into matrix **E** and are not considered here.

The change of the rotationally-symmetrical lens distortion as a function of object distance has been repeatedly investigated ([2], [3], [8], [19]). It has been shown that the rotationally-symmetrical lens distortion for an object at a certain distance can be derived by interpolation from known distortions for two other distances

$$dr_j = a \cdot dr_1 + (1 - a) \cdot dr_2 \tag{11}$$

with

$$a = \frac{s_2 - s_j}{s_2 - s_1} \cdot \frac{s_1 - f'}{s_j - f'} \tag{12}$$

where s_j is the distance to an object in focus and f' the focal length of the camera. Image formation with an ideal lens and paraxial rays is governed by the formula

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f'} \tag{13}$$

where s is the object distance, s' the image distance and f' the focal length. The image of a point is therefore located at a distance

$$s' = \frac{f' \cdot s}{s - f'}$$

and not at f'. If a camera focussed for an object at s_j is used to image an object located at s which differs from s_j, than the image at s' will not be located in the image plane situated at s_j', and a lens distortion correction interpolated for an object at a distance s must be modified for the scale difference caused be the difference in the image distances. Hence, using (11)

$$dr = dr_j \cdot \frac{s'}{s'_j} = (a \cdot dr_1 + (1-a) \cdot dr_2) \cdot \frac{s'}{s'_j}.$$

If the camera has been calibrated for a finite distance s_1 resulting in the lens distortion values dr_1 and for $s_2 = \infty$ resulting in dr_∞ then the factor a defined by (12) becomes

$$a = \frac{s_1 - f'}{s_j - f'},$$

and dr becomes

$$dr = \frac{(s_1 - f') \cdot dr_1 - (s_1 - s_j) \cdot dr_2}{s_j - f'} \cdot \frac{s'}{s'_j}.$$

Assuming further that s_j is of the same order of magnitude as s_1 but significantly larger than f', or that dr_∞ is neglegibly small, one obtains

$$dr = \frac{s_1 - f'}{s_j - f'} \cdot \frac{s'}{s'_j} \cdot dr_1. \tag{14}$$

The difference between the centre of the entrance pupil and the frontal cardinal point results in the so-called pupil aberration which causes according to [19] the following approximate correction to the radial lens distortion:

$$d(dr) = \left(\frac{1}{s'_j} - \frac{1}{s_j}\right) \cdot P \tag{15}$$

with P defined as a polynomial consisting of even-power-of-r and odd-power-of-r terms.

It was shown in [5] that a decentring distortion profile (consisting only of the term $d_1 r^2$ of equation (9)) is also subject to a scale change, if the lens is used at different focus settings. For a focussed-at object located at a distance s_j, the decentring profile P_j becomes

$$P_j = \frac{f'}{s'_j} \cdot P_\infty.$$

The scale factor f'/s'_j can be redefined using equation (13) as

$$\frac{f'}{s'_j} = 1 - \frac{f'}{s_j}.$$

For an object located at s not in the focussed-at plane at s', this scale factor requires modification with the scale factor given in equation (14). Hence,

$$P = \frac{s_1 - f'}{s_j - f'} \cdot \frac{s'}{s'_j} \cdot \left(1 - \frac{f'}{s_j}\right) \cdot P_\infty \tag{16}$$

The image space vector $\mathbf{x}$ is completely defined by equation (4) if the affine lens distortion given by equations (10) and the pupil-aberration distortion given by equation (15) are disregarded: equations (5), (6) and (14) define the rotationally-symmetrical component and equations (7), (8), (9) and (16) the decentring component.

An Algorithm

The definitions for the lens distortion components given in the preceding paragraph still suffer from a lack of clarity in detail: they do not indicate which image-centre reference point(s) (*PBS*, *PPA*, others?) or how many polynomial terms are to be used. In addition, the respective terms of both polynomials ((6) and (9)) are highly correlated. Other correlations may also exist depending upon the geometry of the calibration set-up, as discussed for self-calibration for example in [18] for close-range applications or [20] for aerial photography. It is therefore important that an agreement is reached not only on a detailed mathematical model but also on a strategy or procedure for the use of that model. A procedure consisting of four steps has been reported in [20], where the affine and the pupil-aberration lens distortion components were not considered:

- step 1: determination of either the calibrated or the equivalent focal length using image coordinates already corrected for the rotationally-symmetrical lens distortion of the used lens type,
- step 2: determination of the decentring lens distortion in respect to the *PPA*,
- step 3: determination of a *PBS* and of the actual rotationally-symmetrical lens distortion, and

- step 4: simultaneous verification of both lens distortion components.

A discussion of various definitions given for the nouns "heuristic" and "algorithm" in [14] led to the following definition:

> " An algorithm presumes a problem and a precise step-by-step procedure that solves the problem or shows it insolvable ... within certain resource limits."

It is obvious that a standardization in regard to the determination of the parameters defining the geometric-optical performance of a camera will require an agreement not only on the definition of a mathematical model but also on an algorithm to be used with this model.

Conclusion

Much work still needs to be done within *ISPRS*, and a number of compromises will be required by all organizations involved in camera calibration, before a standard algorithm for the determination of the parameters defining the geometric-optical performance (or the interior orientation) of cameras using central projection can become a reality. It is hoped that all interested parties will become involved in the development of a standard algorithm and of the underlying mathematical model.

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