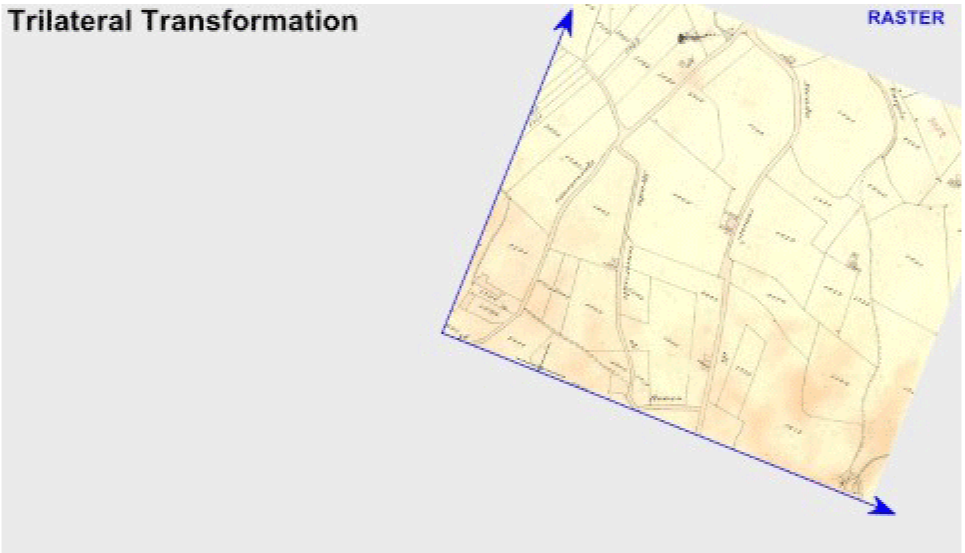
The Trilateral transformation



Although the Homography and Rubber-Sheeting transformations are robust mathematical algorithms, they are not the optimal solution for maps created by a real land survey, because:

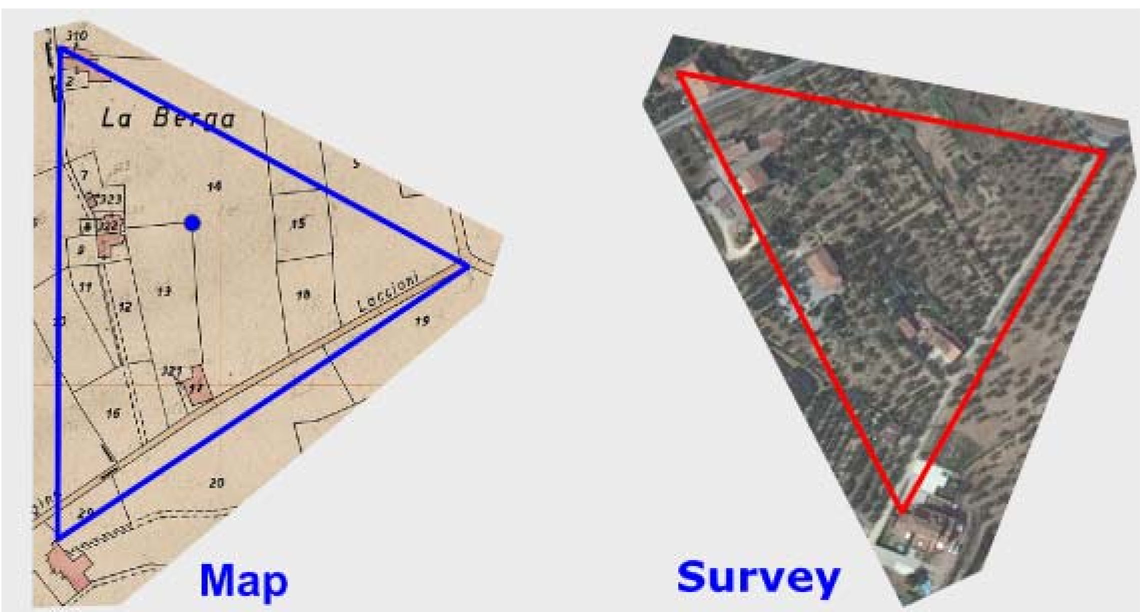


Fig. 1 - The Trilateral transformation relates each triangle on the map to its corresponding triangle on the survey.

- The Homography transformation is based on the assumption that the map is a perspective projection of the terrain. This is the perfect method for maps generated through an aerial photographic survey, but this is not the case of land survey maps

- The Rubber-Sheeting transformation has been studied for adapting the deformed map to a regular grid of control points, but this is not the case when we only have a set of corresponding map-real points and we completely ignore how good the mutual position is for each of them.

The Trilateral transformation does not suffer with these basic conditions because it directly calibrates the map to the survey.

The algorithm firstly applies a triangulation in order to divide the area into single triangles and then it considers each single triangle. Fig. 1 shows the initial situation: given a triangle on the map, we want to relate it to the corresponding triangle on the survey (reality) so that we can calculate the position in the field of a point on the map inside that triangle (the blue spot).

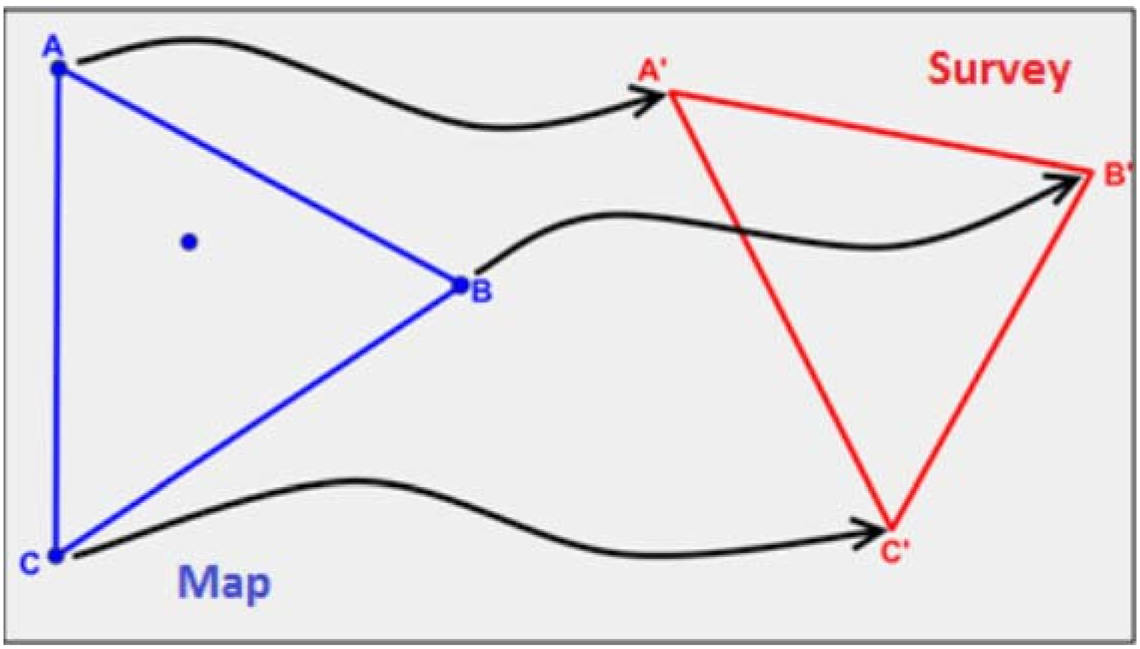


Fig. 2 - Triangle vertices are directly moved from the map to the survey.

How do we achieve this goal? Well, for triangle vertices the solution is straightforward: we simply need to migrate them from the map to their correspondent vertices in the survey as shown in Fig. 2.

For the internal point things are a bit more complicated, but not too much. With reference to Fig. 3, the internal point in the map is projected from a vertex to the opposite side, thus determining the length of the two segments a, b.

Then these two segments are calculated for the survey triangle (a', b') with the same proportion so that the same projection is reproduced in this triangle. Finally, this projection is joined to the corresponding vertex. The operation just described is then repeated for the other two vertices as shown in Fig. 4.

Of course, due to map deformation, the three conjunctions in the survey triangle do not intersect at a single point as in the map triangle, but they form a small internal triangle, which is called the “deformation triangle”. In fact, the size of this internal triangle tells us how big the deformation is: the bigger the triangle, the bigger the deformation is. Finally the map point is moved to the barycenter of the deformation triangle in the survey.

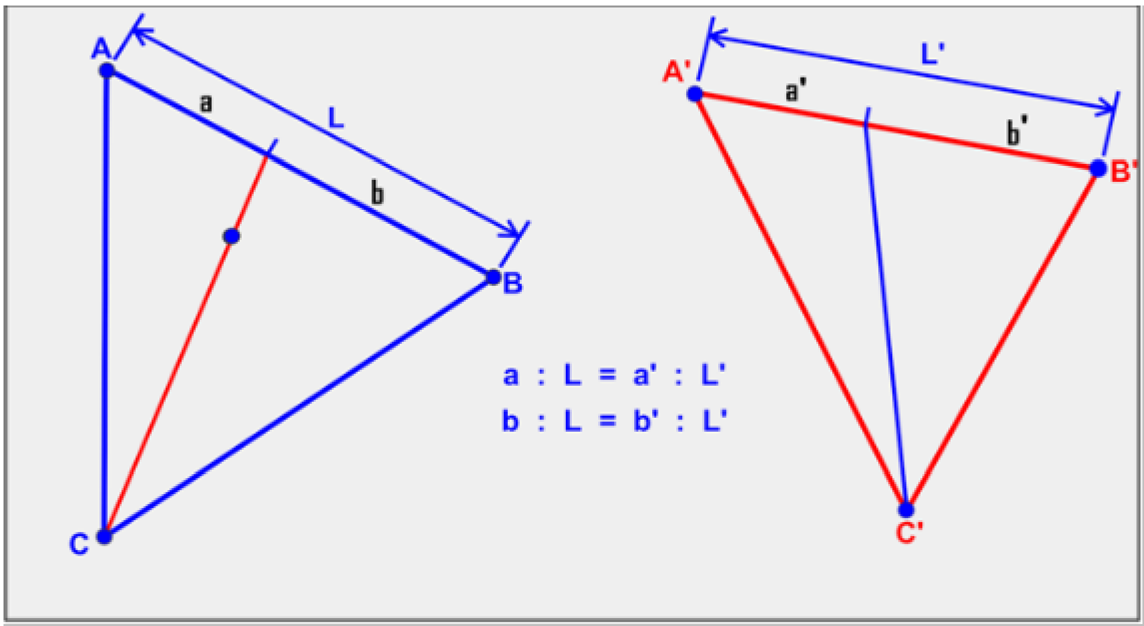


Fig. 3 - The internal point in the map is projected from a vertex to the opposite side, and then the two projections are calculated for the triangle in the survey by applying a simple proportion.

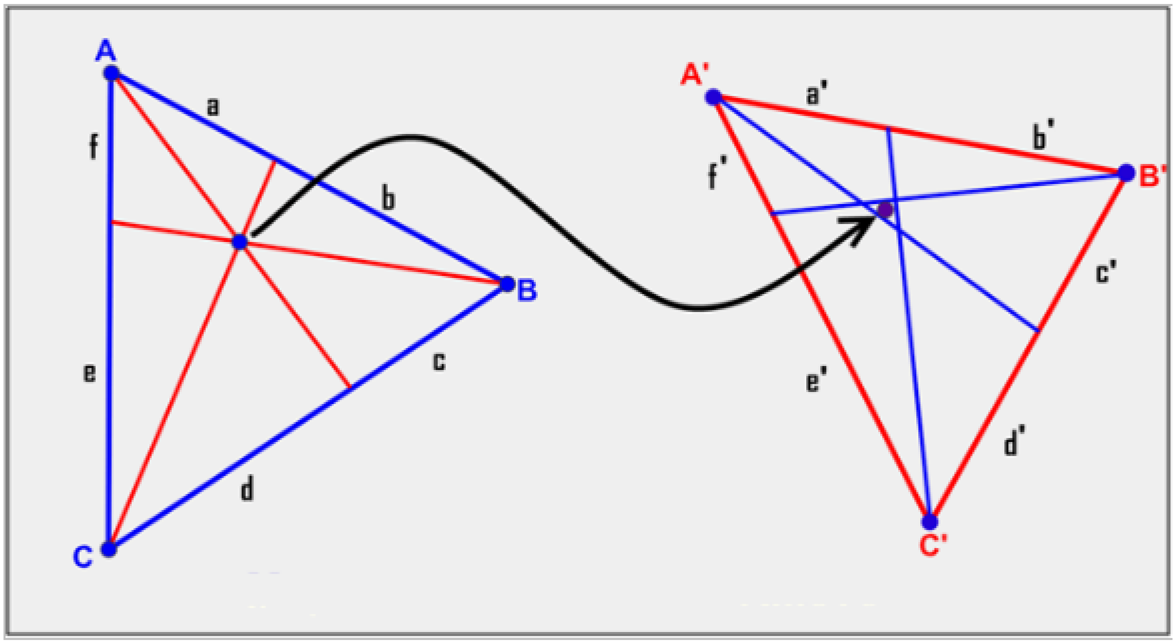


Fig. 4 - The point on the map is moved to the barycenter of the deformation triangle in the survey.