The Grid transformation

In section "Why do raster maps need to be geo-referenced?" we explained that some geo-referencing techniques simply solve 3 problems of a non-geo-referenced map (coordinates, units and orientation) but do not remove the deformation. This is the case of Affine and Barycentric transformations which are therefore not suitable for tasks in which high precision is required.

The Grid transformation instead achieves this result for maps provided with grid lines. These maps can be rectified very accurately by this technique because they can be divided into their single grid squares so that we can consider the non-uniform map deformation.

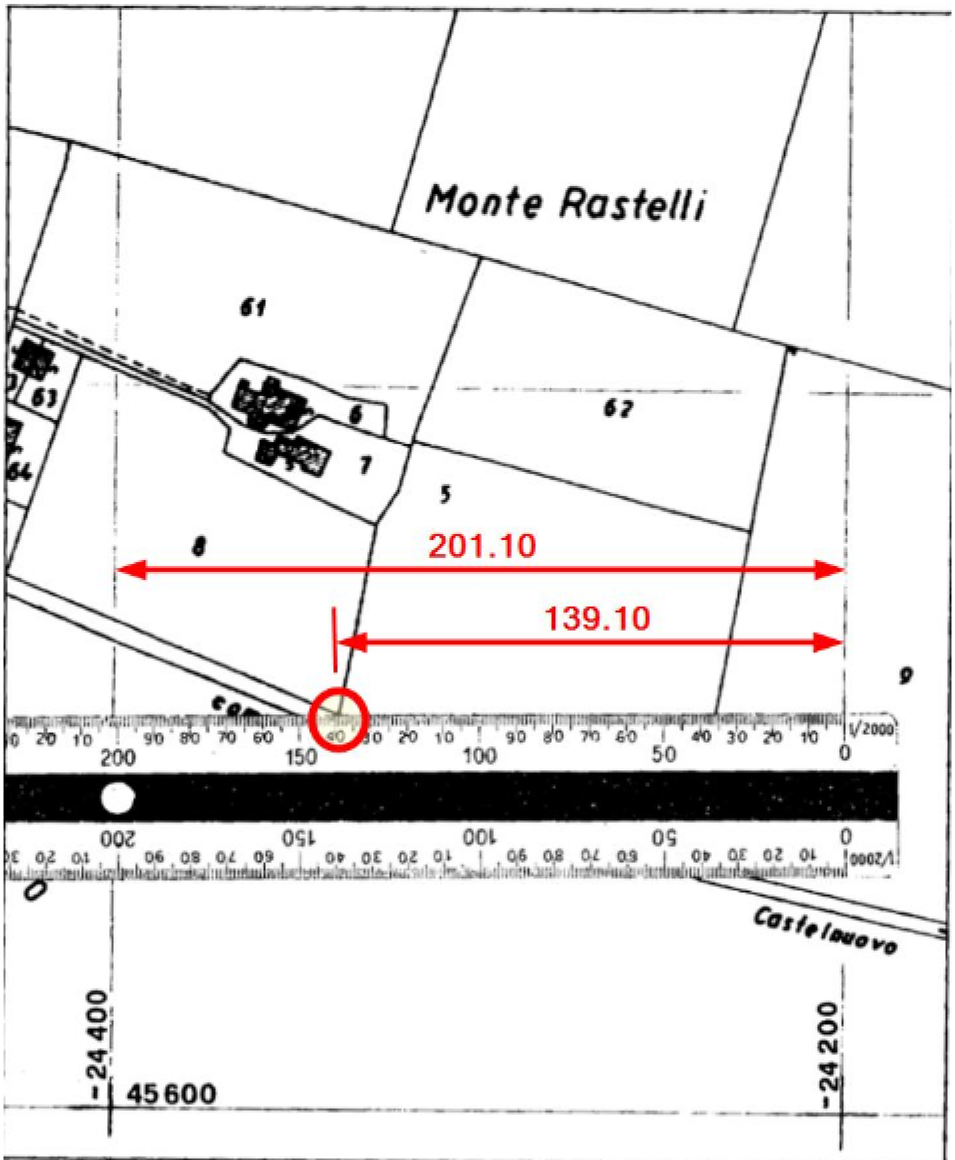

Fig. 1 - The problem of correcting map deformation in a paper map.

In fact, the physical agents deforming the original paper map (heat, humidity, contact with liquids, etc.) might have acted with different degrees in single localized areas of the map. So, if we need to obtain geometric information (coordinates, distances) of a specific localized area of the map, we simply need to correct the deformation of that single localized area.

To explain the Grid transformation, let’s go back to the situation of paper maps at the time when they were not yet available as raster images. How should we act in that case? For example, on the paper map shown in Fig. 1, we want to retrieve the Easting of the point indicated by the red spot.

To do so, we firstly need to verify if the map has been subjected to a deformation in that section. So, with an engineer's scale rule, we measure the distance between the 2 grid lines containing that point. For this map, this distance should be exactly 200 meters, whereas we measure:

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-201.10
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This means that this section is enlarged. To retrieve the correct coordinate, we then need to perform the following calculation:

- Compare the distance retrieved above with the nominal value of the interval and calculate the difference by dividing the two values:

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-200 / -201.10 = 0.9945
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- Measure the point East from the reference grid line and multiply it by the adjusting coefficient found above, hence determining the adjusted distance (deformation removed).

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-139.10 * 0.9945 = -138.34
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- Add the adjusted distance to the absolute East coordinate of the reference grid line.

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-24200 + (-138.34) = -24338.34
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Well, this simple concept of map calibration is also valid for raster maps as well. But of course, as they are available as image files, and using appropriate software, such as CorrMap, we can conveniently add much more precision, rather than just estimating decimals of a millimeter with an engineer's scale rule.

So, following the manual approach described above, the Grid transformation algorithm is based on the assumption that each grid square formed by the grid lines has been subjected to a deformation in both East and North direction. In Fig. 2 and Fig. 3 below we have manually exaggerated the deformation of a square so that the calibration procedure is obvious. With such a magnified deformation we can see that the grid square is no longer a square, but it is quadrilateral because its sides, lying on the grid lines, are no longer parallel.

Therefore, if we need to retrieve the coordinates of a point P inside it (shown by a blue spot in Fig. 2), we should calculate the deformation of the quadrilateral compared to the original square. Fig. 3 shows the geometrical schema of the algorithm. Because the horizontal/vertical grid lines forming the quadrilateral have lost their parallelism, we simply extend them until they intersect in both East and North directions.

Then we conjunct these two intersection points with point P that we are interested in, thus determining the lines AC and DB, which exactly reflect the quadrilateral deformation referred to point P. We then calculate the length of these two segments. If there was no deformation, this length should be exactly 200 meters (for this map) for both AC and DB.

Otherwise different values will tell us how much the original square has deformed in the corresponding direction. In Fig. 3 for example, we found AC = 199 and DB = 201. So what we need to do is to calculate at what coordinates should point P be when the quadrilateral was still a perfect square, with both AC and DB = 200. To do this, the Grid transformation does a reverse-deforming of the quadrilateral until AC and DB both go back to being 200 meters long.

Fig. 2 - The Grid transformation is based on the on the assumption that each grid square has been subjected to a deformation.

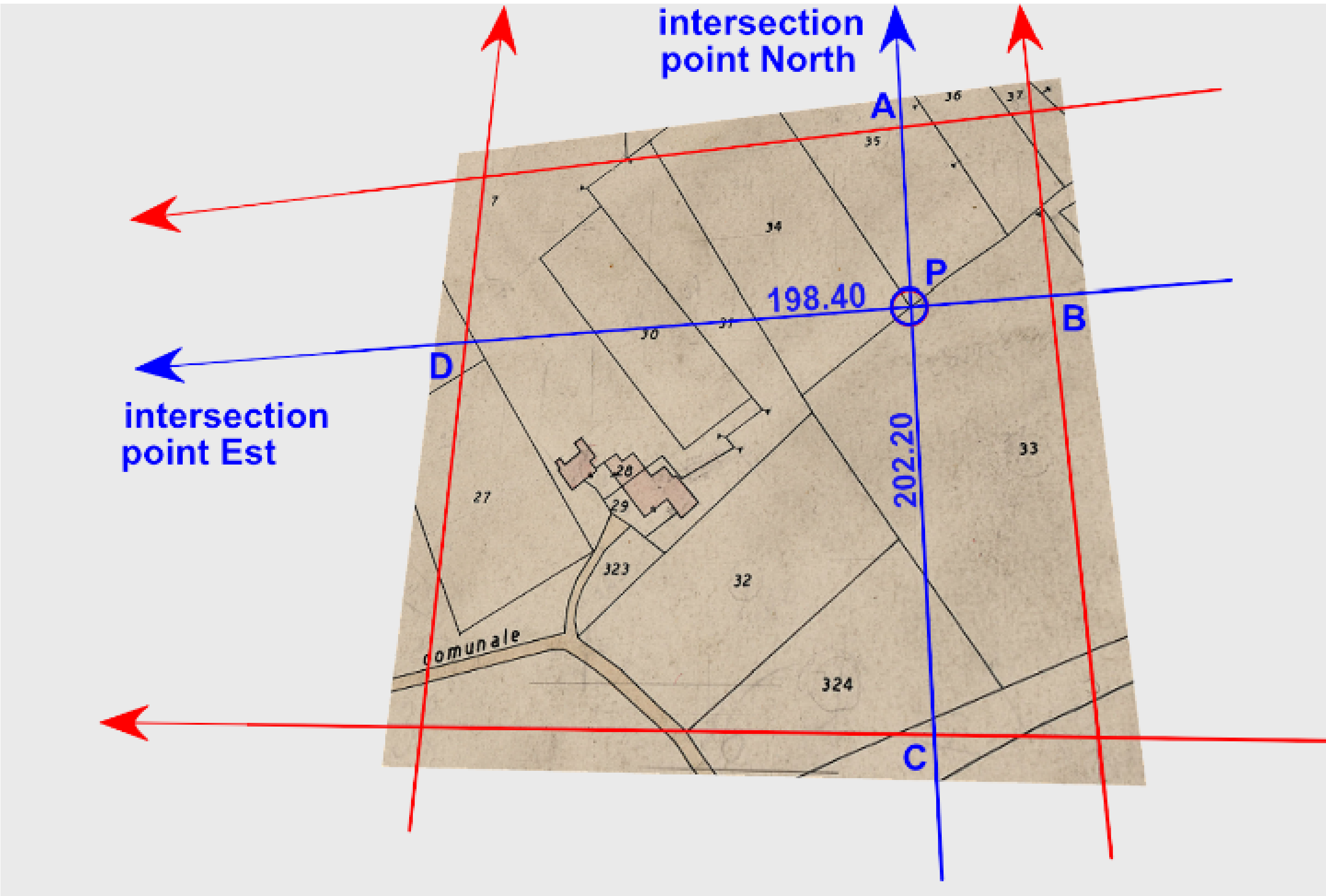

Fig. 3 - The geometrical schema of the Grid transformation.

The sequence of images in Fig. 4 shows this reverse-deformation. The original position of point P is indicated by the blue circle which remains fixed, whereas the red circle shows point P moving until it reaches its final position (box 4).

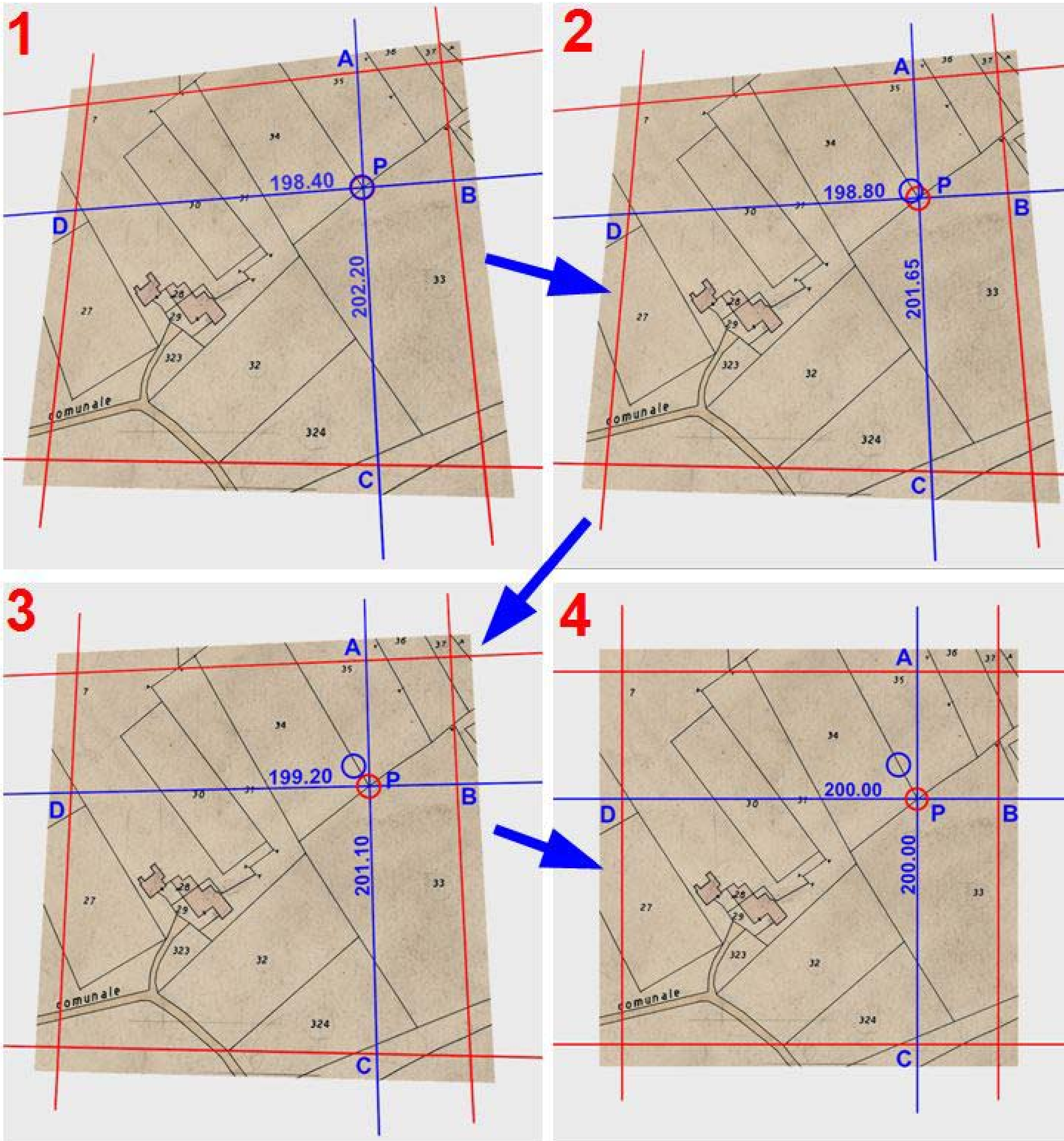

Fig. 4 - The Grid transformation performs a reverse-deformation of the grid squares (quadrilaterals).