

# PRECISE METHOD OF FISHEYE LENS CALIBRATION

Michal Kedzierski, Anna Fryskowska

Dept. of Remote Sensing and Geoinformation, Military University of Technology, Kaliskiego 2, Str. 00-908  
Warsaw, Poland - (mkedzierski, afryskowska)@wat.edu.pl

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### ABSTRACT:

In close-range photogrammetry productivity and accuracy of optical systems is very important. Fisheye lens camera can provide data in hard to reach places or in very small distance to the object. Nevertheless such lenses have a very large distortion. Paper presents new approach to the digital camera calibration process. Proposed method bases on differential geometry and on determination of arc curvature in every segment of photographed test. Making regression of this curve, we obtain straight line and point laying on the straight without distortion. Researches were made using calibration test field containing about 250 reference points and two cameras Kodak DCS 14nPro with 10.5 mm and Nikon 65 with 16 mm fisheye lenses. In a paper there is described conception and results of precise fisheye lens calibration method.

## 1. INTRODUCTION

Fisheye lenses provide imaging a large area of the surrounding space by a single photo, sometimes more than 180 deg. They make possible to realize photo on very small distance, what in some engineering elaboration aspects may be particularly useful. Close range photogrammetry (central perspective) does not comply with fisheye image processing. The fundamental difference between a fisheye lens and classical lens is that the projection from 3D ray to a 2D image position in the fisheye lens is intrinsically non perspective. One fact has to be taken into consideration - that not all fisheye lenses give hemispherical image. In our experiment there were used fisheye lens with focal lens 10.5 mm, which image is not hemispherical. Application of such type of fisheye lens gives more possibilities of usage of images in close range photogrammetry eliminating from the image everything above FOV of 170°, and preventing simultaneously retrieval of image radius. Images were taken with digital camera Kodak DCS 14n Pro f=10.5mm with matrix 4500x3000 pixels.

In order to making our method more reliable, we repeated measurements for second camera with 16 mm lens, mounted in classic camera Nikon 65. In this second case, photos were taken in the B & W film with 50 ISO film speed, and scanned with 2500 dpi resolution.

Fisheye lens has a very large distortion, for which the distortion polynomial used here would not converge. For such a lens the image coordinates should be represented as being ideally proportional to the off-axis angle, instead of the tangent of this angle as it is in the perspective projection. Then, a small distortion could be added on the top of this. Furthermore, the position of the entrance pupil of a fisheye lens varies greatly with the off-axis angle to the object, therefore this variation would be modeled unless all viewed objects are very far away.

The calibration of dioptric camera involves the estimation of an intrinsic matrix from projection model. The intrinsic matrix, which maps the camera coordinates to the image coordinates, is parameterized by principal points, focal length, aspect ratio and skewness.

The projection from 3D rays to 2D image positions in a fisheye lens can be approximated by the imaginary equidistance model. Let a 3D ray from pp of the lens is specified by two angles. Together with the angle  $\phi$  between the light ray reprojected to xy plane and the x axis of the camera centered coordinate system, the distance  $r$  is sufficient to calculate the pixel coordinates:  $u' = (u', v', 1)$  and in some orthogonal image coordinate system, as  $u' = r \cdot \cos\phi$ ;  $v' = r \cdot \sin\phi$ . The complete camera model parameters including extrinsic and intrinsic parameters can be recovered from measured coordinates of calibration points by minimizing an objective function with denotes the Euclidean norm.

A circular fisheye camera is a result of the size of the image plane charged coupled device (CCD) being larger than the image produced by the fisheye lens.

In the experiment, the calibration points (230) were located on 3D test, with an error  $m_{xyz} = \pm 0.0007$  m. The test is painted special super matt paint, precluding light reflections. While lighting the test does not cause any shadows on the test and its background. The picture used in this experiment was taken in the distance of 0.5 m (a depth of the test is 1.5 m). Points of the test are located on the simple metallic elements forming in the space straight segment. Image of these points in the photo is a circular sector on the plane. Because the lens elements of real fisheye lens may deviate from precise radial symmetry and they may be inaccurately positioned causing the fact, that the projection is not exactly radially symmetric, Kannala and Brandt propose adding two distortion terms, in the radial and tangential direction. In our investigations we propose determination of distortion on the basis of proper mathematical relations: between this segment in the space and the arc on the plane.

Using differential geometry we can determine very precisely distortion value in radial and tangential direction. This relations base on determination of arc curvature in every segment of photographed test. In comparison with previous presented by us methods of fisheye lens calibration, application of such a method in determination of radial and tangential distortion, gave

about 30 % increase of its determination.

Possibility of such an approach, gives a construction of this test. It guarantees location of the test points in one vertical axis with 0.5 mm accuracy.

## 2. THEORY

Taking into consideration arc G with class  $C^2$  created from image points laying on the plumb line of the test, we can say that: if tangent in a point M (ordinary point) is not parallel to  $Oy$  axis, arc G in a neighborhood of the point M can be created using equation (1), where function  $y(x)$  has a second continuous derivative.

$$y = y(x) \tag{1}$$

because equation of all circles series we can express by a formula (2),

$$(x - \xi)^2 + (y - \eta)^2 - R^2 = 0 \tag{2}$$

where parameters are center coordinates  $(\xi, \eta)$  and radius R, according to theory of osculating curves, we can claim, that we obtain osculating circle to arc G in point  $M(x_0, y_0)$  by creating function described below (3).

$$\Phi(x) = (x - \xi)^2 + (y(x) - \eta)^2 - R^2 \tag{3}$$

It fulfills conditions:

$$\begin{aligned} \Phi(x_0) &= 0 \\ \Phi'(x_0) &= 0 \\ \Phi''(x_0) &= 0 \end{aligned} \tag{4}$$

If we replace  $x_0$  with  $x$ , the conditions (4) can be written as:

$$\begin{aligned} (x - \xi)^2 + (y - \eta)^2 - R^2 &= 0 \\ x - \xi + y'(y - \eta) &= 0 \\ 1 + y'^2 + y''(y - \eta) &= 0 \end{aligned} \tag{5}$$

from third condition equation (5), calculating  $\eta$ , and from second and first conditions  $\xi$  and R, we obtain equations (6) expressing center coordinates  $(\xi, \eta)$  and R – radius of osculating circle (Biernacki, 1954).

$$\begin{aligned} \xi &= x - y' \frac{1 + y'^2}{y''} \\ \eta &= y + \frac{1 + y'^2}{y''} \\ R &= \frac{(1 + y'^2)^{2/3}}{|y''|} \end{aligned} \tag{6}$$

if curves has mathematical form:  $x=x(t), y=y(t)$ , and taking into account differential calculus and variations of variables, it is clear that:

$$y'(x) = \frac{dy}{dx}, \quad y''(x) = \frac{\frac{dx}{dt} \frac{d^2y}{dt^2} - \frac{dy}{dt} \frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^3} \tag{6a}$$

using formulas above we obtain equations of center coordinates  $(\xi, \eta)$  and radius of osculating circle R as parameterized formulations (Gdowski, 1997).

$$\begin{aligned} \xi &= x - \frac{(x'^2 + y'^2)y'}{x'y'' - x''y'} \\ \eta &= y + \frac{(x'^2 + y'^2)x'}{x'y'' - x''y'} \\ R &= \frac{(x'^2 + y'^2)^{3/2}}{|x'y'' - x''y'|} \end{aligned} \tag{6b}$$

Because radius of curvature is equal to radius of osculating circle we can say that center of circle is a center of curvature. Set of curvature centers is an evolute, and its equation we can express the same as it is in formula (6) (first and second equation).

Geometric interpretation says that: curvature center is a normal point to the curve in the point M, in which curve touches its envelope, and the same it has limited position.

Intersection point of normal in point M with normal in neighboring point  $M_i$  approach this position (Biernacki, 1954).

Taking into consideration that plumb lines of test (in case of fisheye lens) are conic sections (exactly fragments of ellipses) we can describe them by equation (7) or as parametric formulas (7a).

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{7}$$

$$\begin{aligned} x &= a \cos t \\ y &= b \sin t \\ 0 &\leq t < 2\pi \end{aligned} \tag{7a}$$

thus, as it was mentioned, that evolute is identical as envelope of normal, we can derive it from equation of normal (8).

$$y'(x)[Y - y(x)] + X - x = 0 \tag{8}$$

Now, we insert parametric expression of ellipse (7a) into formulas (6b), we obtain:

$$\xi = \frac{c^2}{a^2} \cos^3 t$$

$$\eta = -\frac{c^2}{b^2} \sin^3 t \tag{9x}$$

where  $c^2 = a^2 - b^2$ , and eliminating  $t$ , we get evolute equation (10x) for ellipse.

$$(a\xi)^{2/3} + (b\eta)^{2/3} = c^{4/3} \tag{10x}$$

If we make regression of our curve, expressed by a formula (7), we obtain straight line equation (11) matched to the curve. On the basis of two points: curvature center and curve point describing plumb line of the test, we can determine straight line equation (12).

$$y = ax + b \tag{11}$$

Intersection points of straight line (11) with straight lines created according to equation (12) can determine shifts of curve points (7) (with distortion) on points laying on the straight line without distortion (figure 1).

where:

$$a = \frac{n \sum x_k y_k - \sum x_k \sum y_k}{n \sum x_k^2 - (\sum x_k)^2}$$

$$b = \frac{1}{n} (\sum y_k - a \sum x_k) \tag{11a}$$

$$y = \frac{(Y_k - y_i)}{(X_k - x_i)} x + y_i - \frac{x_i (Y_k - y_i)}{(X_k - x_i)} \tag{12}$$

where:

$$X_k = \xi$$

$$Y_k = \eta$$

Distance between curve point and intersection point (11) and (12) is a distortion value. Distortion is positive, when curvature radius intersect straight line and negative, when radius pass by the straight line.

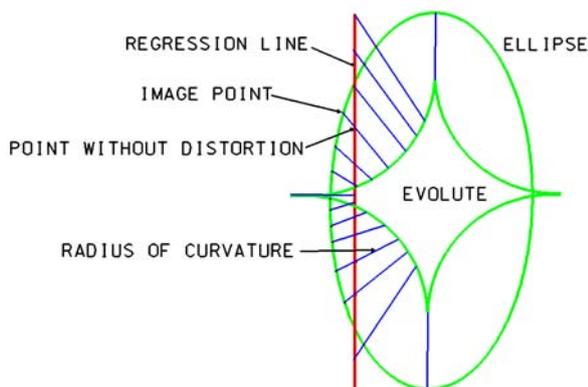


Figure 1. Schema of distortion determination idea.

### 3. EKSPERIMENT

Experiment was done using test field, presented in figure 2.

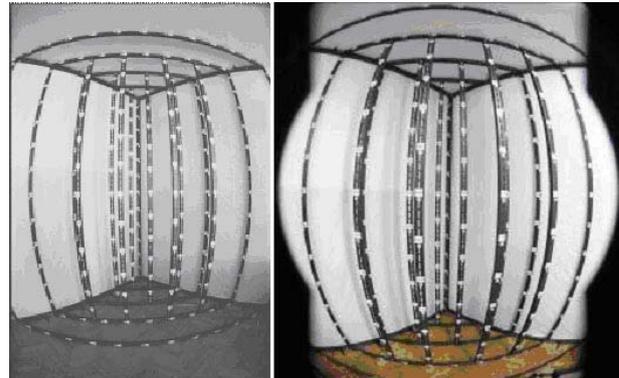


Figure 2. Calibration test; images: on the left: with Nikon 16 mm, on the right Nikon 10.5 mm lens

As the next step, straight line coefficients and statistical analyses were calculated.

Line	Lens 10,5 mm		Lens 16 mm	
	A	b	A	b
56-65	-0,01783	2503,253	-0,02149	3093,321
77-88	-0,0244	442,4195	-0,00466	372,2463
89-100	-0,01073	872,8104	-0,00887	978,3053
101-112	0,007564	1309,888	-0,00993	1494,114
113-124	0,028315	1884,573	-0,01712	2138,627
125-136	0,04134	2267,059	-0,02117	2618,112
149-160	0,001459	1220,161	-0,01194	1366,574
161-172	0,014498	1613,97	-0,01519	1843,48
185-196	0,037156	2192,086	-0,01864	2571,018
197-208	0,016929	1437,506	-0,00691	1626,382
221-232	0,018275	1515,701	-0,00904	1721,471
Mean value	0,010234	1569,039	-0,01318	1802,15
Correlation	-0,45293	0,997954		

Table 1. Coefficients of straight line equation (11) for plumb line in the image.

Correlation coefficient for parameters  $a$  of a straight line equation equals 0.45. It follows from deformation value caused by distortion different for both lenses: 10.5 mm and 16 mm.

Proposed algorithm is perfect for image correction before measurements connected with triangulation or surface model. Figure 3 shows calibration effects.



Figure 3. from the left: original image and image after calibration

	10.5 mm	16 mm	Parameter
C <sub>k</sub>	±15 μm	±17 μm	focal lens
x <sub>o</sub>	±0.7 μm	±0.7 μm	coordinates of the principle point
y <sub>o</sub>	±0.8 μm	±0.7 μm	
k <sub>1</sub>	±6.1·10 <sup>-6</sup>	±5.8·10 <sup>-6</sup>	radial distortion
k <sub>2</sub>	±9.1·10 <sup>-8</sup>	±8.2·10 <sup>-8</sup>	
k <sub>3</sub>	±3.3·10 <sup>-10</sup>	±3.4·10 <sup>-10</sup>	
P <sub>1</sub>	±1.2·10 <sup>-6</sup>	±1.4·10 <sup>-6</sup>	tangential distortion
P <sub>2</sub>	±2.9·10 <sup>-7</sup>	±3.4·10 <sup>-7</sup>	
C <sub>1</sub>	±3.2·10 <sup>-5</sup>	±1.9·10 <sup>-5</sup>	affinity and shear
C <sub>2</sub>	±4.9·10 <sup>-5</sup>	±6.0·10 <sup>-5</sup>	

Table 2. Error of calibration fisheye lens Nikon 10.5 mm and Nikon 16mm

After initial corrections, we have to be measure the images again, and make a process of self-calibration as well. Distortion is eliminated almost completely. Nevertheless, we have to remember that, problems with self-calibration usually apply to parameters correlation. It is so, because bundle adjustment method (by self-calibration) in determination of interior orientation parameters usually causes correlation between adjusted parameters. Therefore, we should avoid of high correlation values, which indicates on linear dependences between particular parameters. Another important element is at least one known distance in imaging direction (for instance known reference coordinates) or one reference coordinates set (for instance oblique photograph of calibration test) (Luhmann, 2006).

#### 4. CONCLUSION

Proposed calibration method of fisheye lens is suitable not only for image corrections, but also for precise data acquisition. It is especially important, when using classical lens with normal focal length is unfounded. In a paper aspect of distortion elimination was more emphasized than classical approach of self-calibration. What has to be mentioned is the fact that in lesser-precise measurements, my method can be finished after distortion correction.

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