

A NEW METHOD FOR VANISHING POINTS DETECTION IN 3D RECONSTRUCTION FROM A SINGLE VIEW

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ABSTRACT:

From a single view that contains a three dimensional scene is possible to obtain substantial information about the object and even its partial reconstruction. This analysis can provide, in the case of Photogrammetry, measures of the original object, and in the case of Computer Vision, useful information to 3D reconstruction. Nevertheless, to carry out this analysis, a critical part of the process is to identify with a high degree of quality, the structural elements of an oblique image, mainly vanishing lines and points. The presence of mini-segments due to the automatism, and the own weakness of the scene's geometry, provide unfavorable intersection cases. Taking into account the qualitative and quantitative gap of complexity associated to real images corresponding to architectural buildings, this paper aims to present a new method for vanishing points detection that takes a scientific approach combining several proven techniques and which applies three different steps:

1. A *Clustering step*, in order to cluster the mini-segments in vanishing lines.
2. An *Estimation step*, which applies a modification of the Gaussian sphere method, in order to obtain an estimation of vanishing points and reject possible erroneous vanishing lines.
3. A *Computation step*, which applies a re-weighted least square adjustment support by an M-estimator.

1. INTRODUCTION

The motivation and usefulness of precise and reliable determination of vanishing points, among other structural elements belonging to the oblique image, has been demonstrated based on their correspondence with the three main orthogonal directions. Particularly, in architectural environments vanishing points provide three independent geometrical constraints which can be exploited in several ways: from the camera self-calibration and a dimensional analysis of the object to its partial 3D reconstruction.

Therefore, nowadays there is a huge proliferation of different methods for vanishing points detection. One of the most renowned methods is the Gaussian Sphere, which holds multitude of variants. Initially, the Gaussian Sphere method is developed by Barnard in 1983 (Barnard, 1983), with the goal of avoiding the singularity generated by the vanishing points situated in the infinite. This approach accomplished by Barnard, is later on improved by Magee and Aggarwal in 1984 (Magee&Aggarwal, 1984), accumulating the projection of intersection points belonging to lineal segments over the Gaussian Sphere. This method carries a considerable computational cost but in turn it is more accurate. At the same time, other authors define new spaces of accumulation. Brillault in 1986 (Brillault, 1986) sets up an uncertainty model for lineal segments. Then, Brauer and Voss in 1999 (Bräuer&Voss, 1999) propose another consistent alternative supported by the normal interpretation planes calculation, while Shufelt introduces an improvement to the Gaussian Sphere method, which integrates the knowledge of basic primitives geometry (rectangular and triangular prisms), that support the search of vanishing points (Shufelt, 1999). A peculiarity among Shufelt and others authors is the application context of his methods. While most of the authors link their methods to applications related to Computer

Vision and Close Range Photogrammetry, Shufelt researches in Aerial Photogrammetry context. Nevertheless, others different approaches to the Gaussian sphere have been proposed: Lutton in 1998 (Lutton, 1998) after analyzing the error due to the limited parameter space of Gaussian sphere, adapts the Hough transform to vanishing points computation. Shortly after, (Tuytelaars et. al., 1998) apply the Hough transform supported by an iterative and hierarchical sequence known as 'Cascaded Hough Transform'. A main drawback of all techniques which transfer information from the image into a bounded space is that the original distances between lines and points are not preserved. This drawback can be avoided when line segments are not transformed into a bounded space, i.e. the image plane is chosen as the accumulator space. In this line, (Quan&Mohr, 1989) design a search step in a straight forward manner. Firstly, the dominant vanishing point, which corresponds to the accumulator cell with the most line segments, is detected. After removing the line segments, which correspond to this vanishing point, the search for a maximum in the accumulator space is repeated. This iterating process stops when the number of line segments of a dominant vanishing point is below a certain threshold. (Rother, 1999) develops a flexible framework, the unbounded image plane, which can be used for the accumulation step as well as for the search step. On the other hand, Frank Van den Heuvel in 1998 (Heuvel, 1998) develops a new variant for vanishing points detection, which presents a hierarchical and robust structure supported by the application of geometrical constraints and statistical tests. Recently (Almansa et. al., 2003) have developed a method for vanishing points detection which does not require any priori assumption (like orthogonality or coplanarity) and combines some of the above approaches (Gaussian sphere and Hough transform), guarantying precision and reliability in the result.

The method presented in this paper takes a scientific approach which combines several proven methods supported by robust and statistical techniques. In this sense, the key differences of this method with respect to previous approaches are the following: (i) an original method for clustering vanishing lines is developed based on a hierarchical process supported by geometrical constraints; (ii) a modification of the Gaussian sphere method based on analytical concepts is developed in order to obtain a first estimation of vanishing points; (iii) a tending toward integration of M-estimators (Domingo, 2000) in an iterative re-weighted least square adjustment is accomplished for the final computation of vanishing points.

2. A NEW METHOD FOR VANISHING POINTS DETECTION

Next, Figure 1 describes the different steps to the new approach for vanishing points detection:

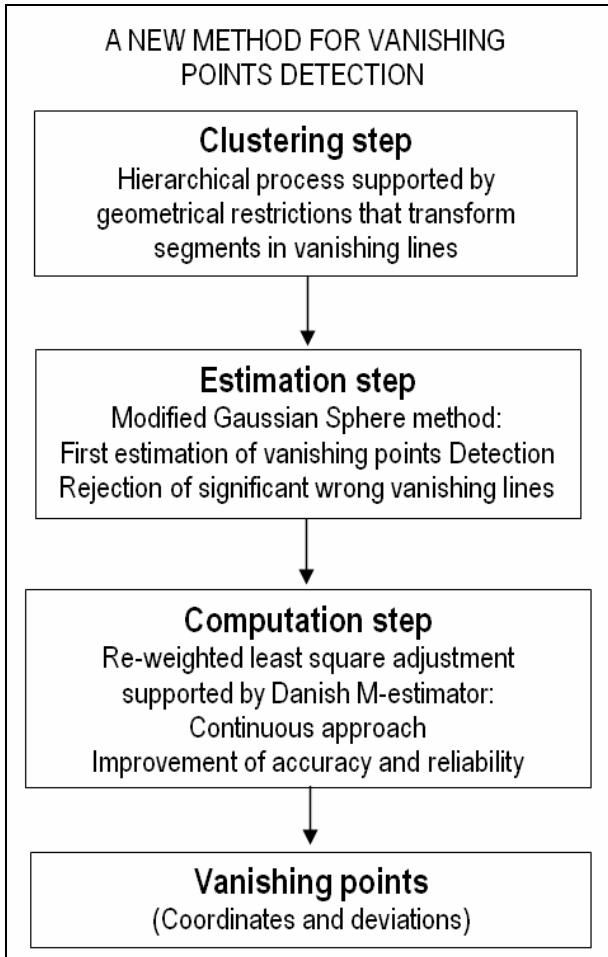


Figure 1. A new method for vanishing points detection

2.1 Clustering step

The clustering step for vanishing lines presents a hierarchical structure divided into two different levels and supported by geometrical constraints.

In the first level, segments are clustered through an iterative process based on their collinearity, taking an orthogonal distance as input parameter or threshold (Figure 2). In this way, for a reference segment, its orthogonal distance with relation to remaining segments is evaluated. Those segments, whose orthogonal distance is lower to the established threshold, will be clustered as collinear segments. Nevertheless, the presence of mini-segments could carry some problems in the clustering process, i.e. leaving unclassified vanishing lines. In this sense, a weight factor for the line coverage has to be considered, which will depend on the number of collinear segments as well as their length. Thus, those segments that have more collinear mini-segments and major length will have priority.

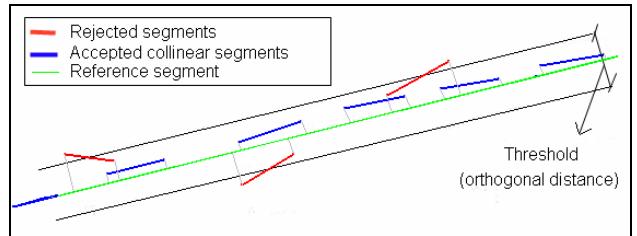


Figure 2. First clustering of segments, collinearity condition

In the second level, collinear segments are clustered according to their correspondence with the three basic vanishing points directions. This second clustering level is based on the RANSAC robust estimator (Fischler&Bolles, 1981). The application of RANSAC together with the consideration of several inherent geometrical constraints in vanishing points (perpendicularity and skyline), allow to carry out a clustering of the segments at the same time that remove erroneous or unnecessary segments for our purposes. In fact, Figure 3 illustrates that the presence of noise due to reflections and shades induce to wrong vanishing lines, which don't correspond with any of the three main directions of the object.

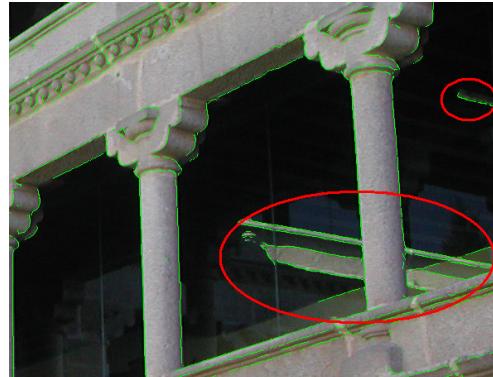


Figure 3. Wrong vanishing lines due to reflections and shades

Particularly, the adaptation of RANSAC to the clustering process consists on the following steps (Figure 4):

1. Random selection of two segments and vanishing points estimation through a simple intersection of lines.
2. Voting process through a segment validation corresponding to a main vanishing direction supported by a slope analysis among the segments and the estimated vanishing point.
3. Repetition a certain number of times the step 1 and 2 (minimum number of random combinations).
4. The three maximum resultant of the voting process after a random number of combinations will correspond with the three main vanishing points and as a result the favorable votes ‘inliers’ of each maximum will contain the vanishing lines coincident with one of the three main directions of the object. The segments that overcome the tolerance of each generated group will be considered as wrong segments ‘outliers’ and will be eliminated.

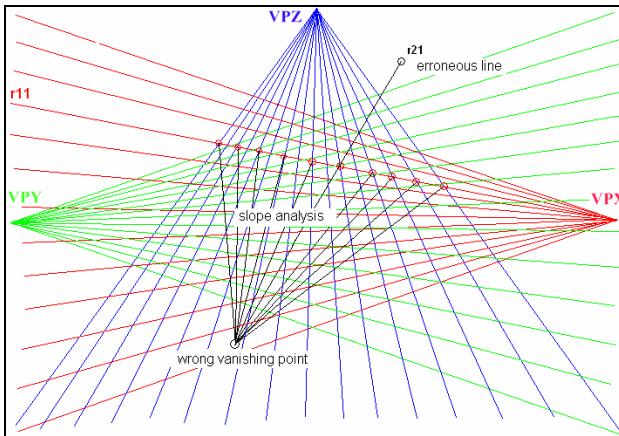


Figure 4. Clustering process based on slope analysis with RANSAC

Nevertheless, the application of RANSAC in clustering process demands a high computational cost. In fact, if we take into account the correspondence between the three basic directions of the object and the three main vanishing groups, and consider certain homogeneity in the number of segments belonging to each group (1/3 of the overall), we will start with a 65.66% of wrong lines ‘outliers’. This percentage translated to RANSAC terminology in number of minimum random combinations would be of approximately 4108 combinations (Hartley&Zisserman, 2000), what could be unapproachable with a high number of segments.

A solution to reduce considerably this number of random combinations consists on applying RANSAC in an adaptative way supported by geometrical constraints. Adaptative RANSAC (Hartley&Zisserman, 2000) enables to modify the

parameters (threshold and number of minimum combinations) in run time.

Regarding to the geometrical constraints imposed in the adaptative RANSAC clustering process:

- Random combinations of segments cannot belong to collinear segments.
- Random combinations of segments will have the same slope sign. In this way, possible clustering errors in segments that present slopes near zero will be avoided.

Concluded the clustering process, vanishing lines will have been classified in three main groups corresponding to the three main directions of the object.

2.2 Estimation step: modified Gaussian sphere method

Estimation step aims to obtain a first estimation of the vanishing points as well as detecting the most relevant wrong vanishing lines. In this way, a different approach to the traditional Gaussian sphere method (Barnard, 1983) based on analytical concepts, interpretation plane¹, have been proposed.

This approach has the following input considerations:

1st consideration: All the interpretation planes pass through the projection center.

2nd consideration: The projection center is considered as origin of the reference system.

From both considerations it is deduced that:

Given a plane defined by the analytic equation: $AX+BY+CZ+D=0$, the independent coefficient D can be considered null ($D=0$). In this way, the equation of the plane is: $AX + BY + CZ = 0$.

In order to get a parametrization of the interpretation planes in a two parameter space (A,B) which enables us to obtain a 2D representation, the following equation (1) will be considered:

$$\frac{A}{C}X + \frac{B}{C}Y + Z = 0 \quad \forall C \neq 0 \quad (1)$$

3rd consideration: Homogeneous coordinates corresponding to each image segment will be considered in the following way: $P_1(x_1,y_1,f)$ and $P_2(x_2,y_2,f)$. The focal f , that represents the distance between the image plane and the Gaussian sphere center, performs as a scale factor. In this way, a spatial reference system (X,Y,Z) whose origin corresponds to the Gaussian sphere projection center, and an image reference system whose origin is situated on the top left corner of the image, will be considered (Figure 5).

¹ The interpretation plane is the basic geometric element in the vanishing point estimation and is constituted for whatever straight segment of the object and the point of view. Obviously, the image segment also belongs to the interpretation plane.

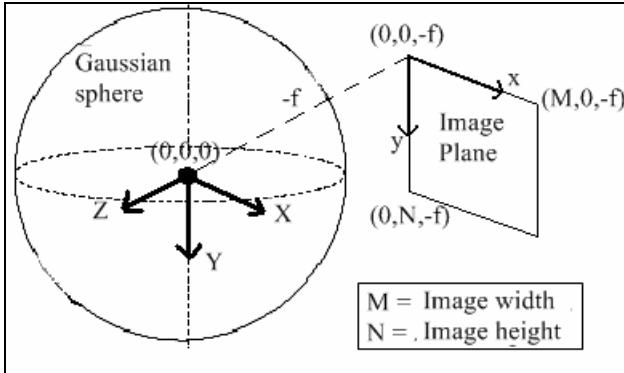


Figure 5. Reference system: Gaussian sphere and image plane

Taking into account the three above considerations, the modified Gaussian sphere method accomplishes the following steps:

1. Initialization of the Gaussian sphere projection center as origin of the reference system (2).

$$\text{Gaussian sphere center : } (0,0,-f) \quad (2)$$

2. Calculation of the interpretation planes for each three non aligned points. Two points belonging to the image segment and one point corresponding to the Gaussian sphere projection center (3).

$$\begin{aligned} (x_1, y_1, z_1) &\rightarrow (x_1, y_1, -f) \\ (x_2, y_2, z_2) &\rightarrow (x_2, y_2, -f) \\ (x_3, y_3, z_3) &\rightarrow (0,0,-f) \end{aligned} \quad (3)$$

3. Parameterization and transformation of the interpretation planes into two parameters A, B (3rd consideration). In this way, each interpretation plane will be represented by a point of coordinates (A, B) and the intersection of several interpretation planes will be represented by the regression line adjusted to these points.
4. Incorporation of a robust procedure supported by RANSAC in order to reject wrong interpretation planes. The adaptation of RANSAC algorithm to the modified Gaussian sphere method is performed following a double algebraic principle in the voting process:

Determinant principle. A cost function is established based on the determinant formed by three planes (4): two interpretation planes randomly selected in each RANSAC combination and a third plane which corresponds with each left over plane. The voting process is supported by the resolution of this determinant. If the result of the determinant is lower (in absolute value) to the fixed threshold², it will denote that the three interpretation planes intersect in a common line, which is a possible

direction of a vanishing point, and therefore a vote will be added. The three major groups of votes will contain the three main directions of the vanishing points.

$$\left| \begin{array}{ccc} A_1 & B_1 & 1 \\ A_2 & B_2 & 1 \\ A_3 & B_3 & 1 \end{array} \right| < \text{Ransac Threshold} \quad (4)$$

Planes pencil principle. Given two random interpretation planes and a third plane corresponding with each one of the left over planes, a voting process will be performed through planes pencil equation (5), in which the sum of the coefficients α and β should be included between the intervals (8) in order to add a vote.

$$\alpha(A_1x + B_1y + z) + \beta(A_2x + B_2y + z) = 0 \quad (5)$$

If an interpretation plane (6) is part of the pencil (5):

$$A_3x + B_3y + z = 0 \quad (6)$$

it has to accomplish the equations system (7):

$$\left| \begin{array}{l} \alpha A_1 + \beta A_2 = A_3 \\ \alpha B_1 + \beta B_2 = B_3 \\ \alpha + \beta = 1 \end{array} \right. \quad (7)$$

with the threshold considered in (8)

$$[1 - \text{Ransac Threshold}] \leq \alpha + \beta \geq [1 + \text{Ransac Threshold}] \quad (8)$$

Based on the above algebraic principles a voting process is set up. The three combinations with the major number of votes will be selected. For each group, all the interpretation planes classified as ‘inliers’ will be considered in the adjustment, those interpretation planes classified as ‘outliers’ will be rejected.

5. Application of a least square regression to the interpretation planes classified as ‘inliers’, so that the three straight lines adjusted will contain the three main directions of the vanishing points (Figure 6).

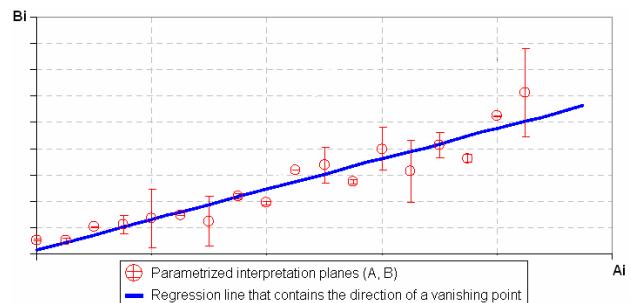


Figure 6. Least square regression corresponding to a vanishing point direction

² Priori deviation of observations will determine the threshold value. This threshold should not be very low in order to maintain relevant information of the object.

6. Finally, an estimation of the three vanishing points through the determination of the intersection between the three

adjusted straight lines and the image plane will be performed. In order to avoid possible singularities, the rank of the determinant (9) constituted by the straight line and the image plane will be analyzed (Burgos, 1993). If the rank of the determinant is 3, an intersection will exist between the straight line and the image plane. Whatever other rank value will denote the non intersection or parallelism between the straight line and the image plane and therefore it will not be considered.

$$\text{rank} \begin{bmatrix} A_1 & B_1 & 1 \\ A_2 & B_2 & 1 \\ 0 & 0 & \frac{D}{C} \end{bmatrix} = 3 \quad (9)$$

where D/C is a constant that defines the image plane according to Figure 5.

2.3 Computation step: re-weighted least square adjustment support by a M-estimator

After detecting the most representative wrong observations, an iterative re-weighted least square adjustment will be applied supported by the triangle area minimization method (Bräuer, 1999) and Danish M-estimator³ (Domingo, 2000). One of the most important advantages of Danish estimator regarding to RANSAC is its continuous approach without total loss of observations, providing more accuracy and reliability in the result.

Particularly, the method consists on minimizing the triangle surface composed by each extracted segment and the corresponding vanishing point, so that the sum of each triangle's area (S) will be minimized (10). Nevertheless, the presence of accidental and 'light' gross errors in observations will carry that each segment does not have the same degree of participation in the adjustment. In this way, a re-weighted least square adjustment support by Danish estimator will be applied. (Figure 7).

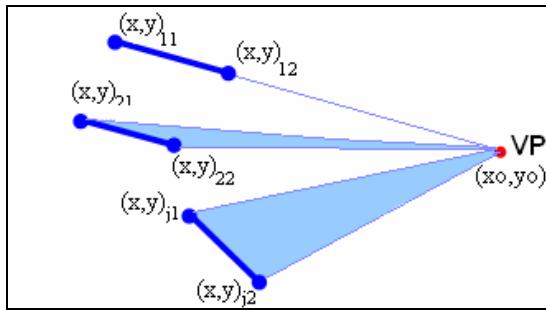


Figure 7. Re-weighted method of triangle area minimization

³ M-estimators are generalizations of the Maximum Likelihood estimators (ML-estimators) and represent an alternative to the classical least square adjustment. In this sense, M-estimators aim to detect and reduce the effect of gross errors, replacing the least square function proposed by Gauss by another different function for the residuals.

$$S^T WS = \sum_{i=1}^n W_i (x_0(y_{i1} - y_{i2}) + y_0(x_{i2} - x_{i1}) + x_{i1}y_{i2} - y_{i1}x_{i2})^2 \quad (10)$$

where W is the weight function based on Danish estimator, (x_0, y_0) the vanishing points coordinates, and $(x_i, y_i; x_2, y_2)$ the image segments coordinates.

The numeric solution for this adjustment follows an iterative re-weighted approach, in which the iteration starts with some initial values for the weights of observations and a conventional least square adjustment. In the following iteration, new weights are calculated for each observation based on the residuals obtained in the previous iteration, and a least square adjustment with these new weights is repeated (11).

$$W(V_i) = e^{(-|V_i|^2)} \quad (11)$$

where V represents the residual vector and i the residual number.

The iterative process continues until the convergence is achieved (usually 3 to 10 iterations).

3. EXPERIMENTAL RESULTS

In order to show the experimental results, two emblematic buildings have been chosen for testing our method, which are situated in Avila (Spain).

Figure 8 illustrates the oblique image corresponding to the first building:



Figure 8. Oblique image corresponding to the first building
Size: (2272 x 1704 pixels)

The input and output parameters corresponding to the clustering step are shown in List 1. Figure 9 illustrates the results.

CLUSTERING STEP

Input parameters:

- Adaptative RANSAC based on slope analysis, with a minimum initial number of combinations of 20, an initial threshold of 0.02 radians and adaptative increment of 0.1 radians, as well as several geometrical constraints.

Output parameters:

- Number of segments clustered in X direction: 389
- Number of segments clustered in Y direction: 402
- Number of segments clustered in Z direction: 99
- Number of segments clustered in another direction: 353

List 1. Parameters of the clustering step

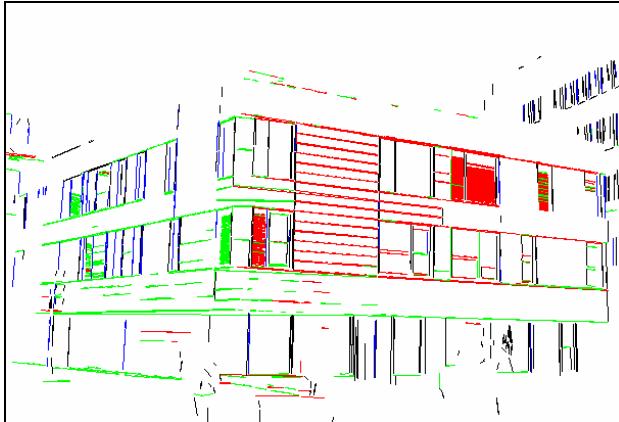


Figure 9. Image corresponding to the vanishing lines clustering

The parameters and results corresponding to the vanishing points detection are shown in List 2 and Table 1 respectively.

MODIFIED GAUSSIAN SPHERE METHOD

Input parameters:

- Number of random combinations: 20
- Threshold: 0.1

Output parameters: Vanishing lines detected as ‘outliers’ in each one of the three directions.

- Number of rejected vanishing lines in X direction: 78
- Number of rejected vanishing lines in Y direction: 193
- Number of rejected vanishing lines in Z direction: 49

List 2. Parameters of the modified Gaussian sphere method

On the other hand, the next graph (Figure 10) illustrates the result of the least squares regression for the three main vanishing points directions.

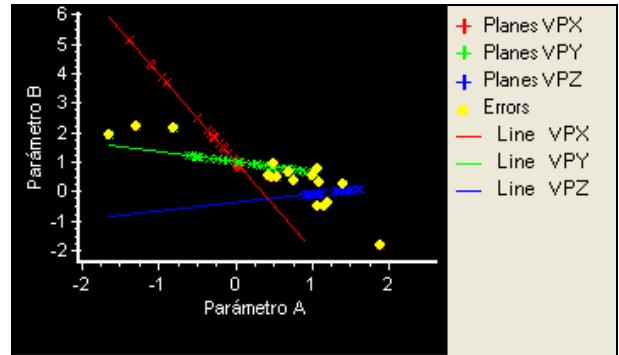


Figure 10. Bidimensional parametrized representation of the interpretation planes

Finally, the results corresponding to the computation of vanishing points after applying a re-weighted least square adjustment based on Danish estimator are shown in Table 1.

Area minimization + Danish estimator (5º iteration)

Area minimization + Danish estimator (Unit: millimeters)	VPX	VPY	VPZ
x (mm)	19.311	-3.414	4.667
y (mm)	4.473	3.989	-65.149
σ_{xx} (mm)	0.064	0.039	0.001
σ_{yy} (mm)	0.001	0	0.117

Table 1. Coordinates (x,y) and standard deviations (σ_{xx}, σ_{yy}) of vanishing points

Figure 11 illustrates the oblique image corresponding to the second building:



Figure 11. Oblique image corresponding to the second building
Size: (2272 x 1704 pixels)

The input and output parameters corresponding to the clustering step are shown in List 3. Figure 12 illustrates the results.

CLUSTERING STEP

Input parameters:

- Adaptative RANSAC based on slope analysis, with a minimum initial number of combinations of 10, an initial threshold of 0.05 radians and adaptative increment of 0.25 radians, as well as several geometrical constraints.

Output parameters:

- Number of segments clustered in X direction: 231
- Number of segments clustered in Y direction: 274
- Number of segments clustered in Z direction: 35
- Number of segments clustered in another direction: 91

List 3. Parameters of the clustering step

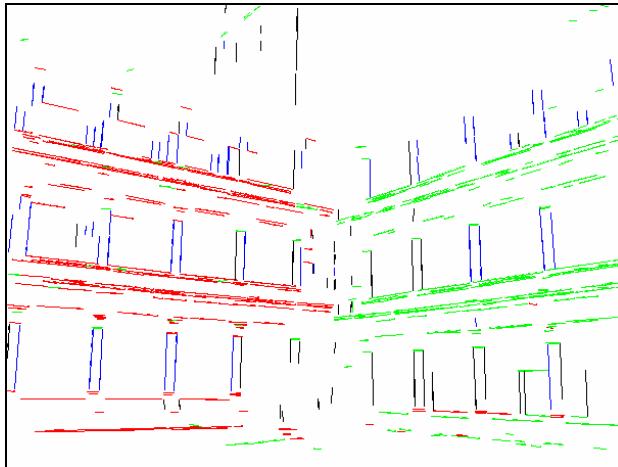


Figure 12. Image corresponding to the vanishing lines clustering

The parameters and results corresponding to the vanishing points detection are shown in List 4 and Table 2 respectively.

MODIFIED GAUSSIAN SPHERE METHOD

Input parameters:

- Number of random combinations: 50
- Threshold: 0.1

Output parameters: Vanishing lines detected as ‘outliers’ in each one of the three directions.

- Number of rejected vanishing lines in X direction: 81
- Number of rejected vanishing lines in Y direction: 72
- Number of rejected vanishing lines in Z direction: 13

List 4. Parameters of the modified Gaussian sphere method

On the other hand, the next graph (Figure 13) illustrates the result of the least squares regression for the three main vanishing points directions.

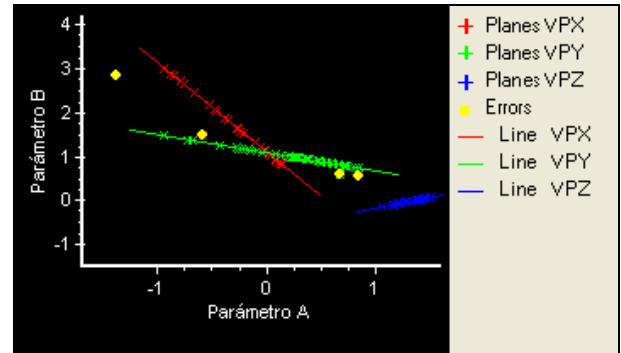


Figure 13. Bidimensional parametrized representation of the interpretation planes

Finally, the results corresponding to the computation of vanishing points after applying a re-weighted least square adjustment based on Danish estimator are shown in Table 2.

Area minimization + Danish estimator (4° iteration)

Area minimization + Danish estimator (Unit: millimeters)	VPX	VPY	VPZ
x (mm)	14.901	-2.018	3.791
y (mm)	4.472	4.176	-30.656
σ_{xx} (mm)	0.054	0.070	-0.004
σ_{yy} (mm)	0.001	0.003	0.093

Table 2. Coordinates (x,y) and standard deviations (σ_{xx}, σ_{yy}) of vanishing points

4. CONCLUSIONS AND FUTURE WORK

A new tendency in vanishing points detection seems to extend towards the development of methods which exploit the redundancy of the observations supported by robust and statistical techniques. Nevertheless, it emerges a double requirement, *automatism-reliability*, seemingly contradictory. While the automatism performs as an incessant source of gross errors, it is the own redundancy inherent to the automatism the key for guaranteeing reliability in vanishing points detection.

The method for vanishing points detection presented in this paper puts together several scientific approaches combining robust estimators. The conclusions extracted in each step are the following:

In the *clustering step*, the algorithm developed has performed efficiently in all cases. Nevertheless, the vertical vanishing lines (Z) belonging to both buildings denote geometric weakness. Consequently, the input clustering parameters (orthogonal distance, slope threshold and number of combinations) have manifested an enormous sensibility, so whatever light variation in the parameters is crucial in order to guarantee accuracy in vanishing points.

In the *estimation step*, the modification of the Gaussian sphere method developed based on the interpretation plane and combining RANSAC robust estimator, has provided a reliable and efficient method for detecting the most relevant wrong vanishing lines and for obtaining a first preliminary estimation of vanishing points. Nevertheless, sometimes the requirement of fixing a threshold manually by the user is a disadvantage to take into account.

In the *computation step*, the application of Danish estimator together with triangle area minimization method has shown a better performance than classical least square adjustment in presence of multiples gross errors, allowing an efficient adjustment (less iterations and unequivocal identification of each error), and a continuous approach (each observation is re-weighted and not directly removed). Consequently, Danish estimator constitutes a good complement for vanishing points methods, mainly when the redundancy of the adjustment is high.

Finally, regarding to future perspectives, the possibility to work with complex scenarios in presence of multiple vanishings points, opens new research lines towards the development of new clustering strategies, as well as multiple vanishing points detection methods. On the other hand, exploit the images provide by security cameras in public places through the detection of vanishing points in order to obtain a metric description of the scene which attends to different studies, i.e. people behavior, is another interesting research line in which we are working now.

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