LENS DISTORTION FOR CLOSE RANGE PHOTOGRAMMETRY

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ABSTRACT

A brief review of formula for radial and decentering lens distortion is presented, with emphasis on the currently accepted representation of the decentering distortion. Recent technological advances, including the automated precision monocomparator, Auto-Set, led to a re-investigation of the analytical plumb-line method of lens calibration. Brown's (1972) extension of Magill's formula for the variation of radial distortion with focussing is experimentally re-verified with data of much higher precision than in previous investigations. The quality of the data led to the finding that decentering distortion also varies with focussing. A re-investigation of the theory of decentering distortion extends the 1966 formulation by Brown to account for the variation experimentally observed.

INTRODUCTION

The theory and formulation of radial and decentering lens distortions adopted for use by the photogrammetric community in the last two decades can be traced to papers by Brown (1966) and Brown (1971, 1972). The first paper demonstrated the projective equivalence of the then widely accepted thin prism model for decentering distortion with the more rigorous and apparently contradictory formulation of Conrady (1919). In Conrady's model the radial component of decentering distortion is three times as large as for the thin prism model. However an appropriate tilt of the camera axis accompanied by a shift in the principal point effectively compensates for the deficiency of the thin prism model. Conrady's model has displaced the thin prism model as the generally accepted model for decentering distortion.

According to Conrady's model, the radial and tangential components of decentering distortion Δr, Δt of an image with coordinates x, y (referred to the principal point as origin) may be represented by the expressions

$$\Delta r = 3P(r)\ \sin(\phi - \phi_0); \quad \Delta t = P(r)\ \cos(\phi - \phi_0) \qquad (1)$$

in which ϕ is the angle between the positive x axis and the radius vector to the image point, r is the radial distance, and the phase angle ϕ_0 represents the angle between the x axis and the axis of the maximum tangential distortion. The profile function P(r) is defined as

$$P(r) = J_1 r^2 + J_2 r^4 + \ldots \qquad (2)$$

where J_1, J_2 are the coefficients of the profile function. In terms of x, y components (Δx, Δy), equations (1) are equivalent to

$$\Delta x = [P_1(r^2 + 2x^2) + 2P_2 xy][1 + P_3 r^2 + \ldots]$$

$$\Delta y = [P_2(r^2 + 2y^2) + 2P_1 xy][1 + P_3 r^2 + \ldots] \qquad (3)$$

The higher order terms (J_2, P_3 ...) are rarely significant and equation (3) can be reduced to linear function of P_1 and P_2. The parameters P_1 and P_2 are related to J_1 and ϕ_0 by the expressions

$$P_1 = -J_1 \sin \phi_0 \qquad J_1 = (P_1{}^2 + P_2{}^2)^{1/2}$$

$$P_2 = J_1 \cos \phi_0 \quad \tan \phi_0 = -\frac{P_1}{P_2} \tag{4}$$

The above model for decentering distortion applies for the lens focussed at infinity. It will be shown in due course that a simple modification will extend its applicability to focussing at finite distances.

In Brown (1971 and 1972) it is shown that for the highest accuracies in close range photogrammetry it is necessary to account for variation of radial lens distortion within the photographic field. The radial lens distortion profile for a lens focussed at a distance s is of the form

$$dr_s = K_{1s}r^3 + K_{2s}r^5 + K_{3s}r^7 + \ldots \tag{5}$$

and the corrections δx, δy to the image coordinates x,y for points in the focussed plane are

$$\delta x = \frac{x}{r} dr_s; \qquad \delta y = \frac{y}{r} dr_s \tag{6}$$

The precise dependence of the coefficients K_{1s}, K_{2s}, ... on s is developed in the above references, as are further modifications appropriate for points not lying within the plane of focus.

The analytical plumb-line method (Brown, 1971) was developed as a rapid, practical way of computing lens distortion parameters at a range of magnifications from approximately 5X to 20X. The principle of this technique lies in the truism that straight lines in object space should project through a perfect lens as straight line images. Any variation from straightness is attributed to radial or decentering distortion, and a least squares adjustment is performed to determine the distortion parameters K_1, K_2, K_3, P_1 and P_2.

The results of such a lens calibration can be contaminated in practice by uncorrected systematic errors in the comparator, by unflatness of the photographic surface or by uncompensated deformations of film. Such sources of error must be adequately controlled or corrected if nonlinearity in measured images of plumb-lines is to be fully attributed to the lens.

Calibrations of radial distortion performed in the late 1960s typically had one sigma values of about two micrometers, with rms values of plate residuals being of similar magnitude. This level of precision was sufficient to confirm the variation of radial distortion within the field of view and with camera focussing. The plumb-line calibrations also produced values for P_1 and P_2 that allowed J_1 and the tangential distortion profile to be calculated with a standard error of a few micrometers. Insufficient evidence was obtained to draw firm conclusions concerning the precise behaviour of decentering distortion with varying object distances.

Analytical Plumb-Line Calibrations in the Mid-1980s.

Some 15 years after the initial plumb-line calibrations reported in Brown (1971) several advances in technology make the plumb-line method an attractive alternative to the (now) usual process of self-calibration for determining lens distortion. These technological improvements include:

The large format 23x23 cm close range camera CRC-1 developed by Geodetic Services Inc. (Brown, 1984). This camera has a vacuum platen flat to within three micrometers and incorporates a set of 25 back-projected reseau targets which allow rigorous compensation for film deformation.

- Utilization of Kodak Technical Pan 2415 film, a very fine grain, black and white film providing the superior image resolution and contrast required for ultra precise applications.

- The development of AutoSet-1, also by Geodetic Services Inc., a fully automatic, computer controlled monocomparator with resolution of 0.1 μm and rms repeatibility and calibrated accuracy of 0.3 to 0.4 μm. AutoSet has automatic line following capabilities ideally suited to mensuration of images of plumb lines.

With such equipment, systematic errors are suppressed to the extent that nonlinearity of plumb-line images is almost entirely attributable to distortion of the lens.

With AutoSet the process of setting on images does not involve the exercise of human skill. Instead, this is accomplished automatically by means of digital image processing of a video scan. As a consequence plumb-lines no longer need be very fine threads (0.3 to 0.4 mm diameter) in order to generate images optimized for human setting. In the tests described in this paper, lines of white translucent monofilament nylon cord of 1.25mm (nominal) diameter were stretched reasonably vertically from ceiling to floor and maintained taut by means of turnbuckles. Measurements at several intervals showed rms variability of cord diameter to 0.010mm. Although these lines are not strictly plumb-lines, they shall be referred to as such for convenience. This is permissible because in the version of the plumb-line calibration exercised, only linearity (and not parallelism) of the lines is assumed.

Twenty-five nominally vertical lines, each 3.0 m long and 150mm apart, were set up in a plane just in front of a matte-black background. Lighting for the lines was provided by fluorescent light tubes placed around the periphery of the calibration range. The lighting was most intense at the edges, thereby counteracting the natural fall-off of light toward the edge of the field of view of a lens.

For each given focal setting of the lens two exposures were made on separate frames, the second with the camera rolled nominally 90° about its axis. Separate frames were used, rather than make both exposures on a common frame, because intersections of grid lines could potentially interfere with the automatic line following operation of AutoSet. Exposure was generally two seconds at f/32.

The initial phase of the monocomparator readings of the film consists of automatic searching and mensuration of the 4 fiducial and 25 reseau marks, the calibrated values of which reside in the AutoSet software. This operation takes less than 60 seconds to complete. A television monitor displays the process to the operator.

After initialization, the operator switches AutoSet to the line following mode. By placing the cursor on the beginning of a plumb-line image and requesting a data point every, say, 4mm, points are automatically recorded along the entire length of the line at a rate of one point per second.

Thirteen well spaced plumb-lines per photograph are usually followed in this manner, producing a data set of about 1300 points per pair of 'horizontal' and 'vertical' line traces. Less than one hour is required for this automated procedure. This compares most favorably with the five or so hours required for the 300-odd points manually observed, to a precision of two micrometers, with double settings on a Mann comparator as reported in the papers cited earlier (for example, Brown, 1971, p.862).

The diameter of the nylon monofilament was chosen to provide optimum images for the automatic target centering software of AutoSet. At a photographic scale of 1:10 a section of plumb-line approximately 180 micrometers long and 125 micrometers wide on the image is scanned to find the centroid. With an effective pixel size of 3.1 by 2.5 micrometers about 3000 pixels are digitized. When such a large pixel population is available, the centroid of a target can be located to considerably better than a tenth of a pixel.

firm conclusions could be drawn because of the uncertainty of the results and the limited sample of lenses.

After several camera/lens calibrations had been performed in 1985 the trend of decentering distortion varying with focussing was again observed, but this time the standard error of the tangential profile ($J_1 r^2$) was well below one micrometer. Consequently, a body of results emerged of sufficient sensitivity to define the behavior of decentering distortion with change in focus. This led to the development of the mathematical model presented below.

The phase angle of decentering distortion (ϕ_0) should be invariant with focussing. The actual variability found in Table 1 is higher than warranted by the standard errors of the individual determinations. Why this should be is unknown at present, but similar excessive variability of ϕ_0 has been found to be the rule more than the exception (for example, see Fritz and Schmid, 1973). However, no systematic variation in ϕ_0 with focussing has been observed.

Formula for Decentering Distortion.

A re-examination of the history of the decentering distortion showed that the thin prism model had been accepted by the photogrammetric community for 40 years (for example, Bennett, 1927) until Brown (1966) proved it to be projectively equivalent to the Conrady (1919) model. Explicit in the mathematical development of the thin prism distortion model is the principal distance 'c', which is used as a scaling factor for the decentering distortion profile (see Brown, 1966, p.446). Upon conversion to the Conrady model (equations (3), (4)) there is no symbolic portrayal of the principal distance, <u>yet it is implicit to the definition of J_1</u>.

The use of equations (3), (4) for the calculation of the decentering distortion when the camera is not focussed at infinity is not in strict conformance with the basic assumptions implicit in their derivation.

To clarify the situation, let the values of the decentering distortion that correspond to infinity focus (principal distance c) be designated as P_1, P_2 and J_1 and let c_s denote the principal distance corresponding to a focussing distance s. From the rigorous derivation of the thin prism model and its projective equivalence to the Conrady model, it follows that

$$P_{1s} = \frac{c}{c_s} P_1 \qquad P_{2s} = \frac{c}{c_s} P_2$$

$$J_{1s} = (P_{1s}^2 + P_{2s}^2)^{\frac{1}{2}} = \frac{c}{c_s} J_1 \tag{7}$$

The principal distance c_s may be designated as $c + \Delta c_s$ where Δc_s is the focussing increment which by virtue of the thin lens formula is related to c and s by

$$\Delta c_s = \frac{c^2}{s-c} \tag{8}$$

This causes the expression $\frac{c}{c_s}$ in equation (7) to reduce to

$$\frac{c}{c_s} = (1 - \frac{c}{s}) \tag{9}$$

Transposition of equations (7) permits coefficients of decentering distortion at infinity to be computed from those corresponding to the focussing distance s.

If the values of decentering distortion at infinity focus are to be chosen as the "standard" and designated as P_1, P_2 and J_1, it follows that the expressions for the decentering distortion Δx_s, Δy_s at an image point x, y can be represented more generally by

In the present investigation results are based on four times more data points than previously. This combined increase in precision and density of data serves to define more clearly the behaviour of decentering distortion with varying object distance. It serves also to provide further and more precise validation of the model for variation of radial distortion with focussing.

Results, Radial Distortion.

The plumb-line range described in the preceding section was used to calibrate several camera/lens combinations in early 1985. The results of one of these calibrations is reproduced in Table 1 and illustrated in Figure 1. Particularly noteworthy in

TABLE 1. EXAMPLE OF RESULTS OF ANALYTICAL PLUMB-LINE CALIBRATIONS. STANDARD ERRORS IN PARENTHESES WERE EXTRACTED FROM COVARIANCE MATRIX OBTAINED FROM INVERSE OF NORMAL EQUATIONS. RMS ERRORS IN FINAL COLUMNS REFER TO RESIDUALS OF *x,y* MEASUREMENTS RESULTING FROM LEAST SQUARES ADJUSTMENT.

Camera: CRC-102 240-mm *f*/9 Fujinon-AS Lens.										
Distance Camera-Plumb-Lines	Photo	Distortion Parameters (and Standard Errors)							RMS Errors (Micrometres)	
		Radial			Decentering					
(metres)	Scale	$K_1(10^{-7})$	$K_2(10^{-12})$	$K_3(10^{-18})$	$P_1(10^{-5})$	$P_2(10^{-5})$	$J_1(10^{-5})$	ϕ_0 (deg.)	x	y
2.64	1:10	−0.662 (±0.007)	0.467 (±0.027)	0.554 (±0.134)	−0.154 (±0.002)	0.066 (±0.002)	0.168 (±0.002)	66.7 (±0.5)	1.0	0.9
3.84	1.15	−0.710 (±0.009)	0.528 (±0.037)	−0.544 (±0.133)	−0.164 (±0.002)	0.082 (±0.002)	0.184 (±0.003)	63.5 (±0.7)	0.9	0.9
5.04	1:20	−0.596 (±0.013)	−0.636 (±0.083)	−0.102 (±0.100)	−0.158 (±0.003)	0.108 (±0.003)	0.191 (±0.004)	55.5 (±0.9)	0.6	0.7

Table 1 are the low magnitudes of the rms values of the x, y residuals of the measured coordinates. These are seen to range between 0.6 and 1.0 micrometers with the larger values corresponding to the larger photographic scales where slight physical irregularities in the nylon cord may amount to a sizable fraction of a micrometer.

To provide further verification of the exactness of Magill's (1955) formula as extended by Brown (1972) to account for the variation of radial distortion with focussing, the following calculation was performed and shown in Table 2. The observed values

TABLE 2. VERIFICATION OF THE MODIFIED MAGILL (1955) FORMULA FOR THE VARIATION OF RADIAL DISTORTION WITH SCALE.

Camera: CRC-010 240-mm *f*/9 Fujinon-AS Lens.					
	Observed Radial Distortion			Predicted 1:15	Difference
Radial Distance	1:10	1:15	1:20	Radial Distortion Using 1:10 and 1:20	Observed-Predicted
(1)	(2)	(3)	(4)	(5)	(3) − (5)
r mm	δr μm	δr μm	δr μm	δr μm	μm
20	−0.4	−0.4	−0.5	−0.4	0.0
40	−3.2	−3.4	−3.7	−3.5	+0.1
60	−10.5	−11.6	−12.5	−11.8	+0.2
80	−24.5	−27.4	−29.5	−27.7	+0.3
100	−46.9	−53.3	−57.3	−53.5	+0.2
120	−78.9	−91.8	−98.1	−91.1	−0.7

of radial distortion for scales of 1:10, 1:15 and 1:20 using a 240mm f-9 Fujinon-AS lens in camera CRC-010 are listed to a radial distance of 120mm. The values for 1:15 scale are then compared to a set of values derived from the use of the 1:10 and 1:20 radial distortion coefficients using equations (1)-(4) from Brown (1972). The discrepancies between the predicted and actually observed values are always less than 0.7 micrometers.

Results, Decentering Distortion.

Brown (1966) produced the currently accepted form of the model for decentering distortion (see equations (3), (4)) and used it over the following years to compute profiles of decentering distortion for a variety of cameras/lenses by means of stellar and plumb-line calibrations. Stellar calibrations obviously employed cameras focussed at infinity, but one of the plumb-line calibrations indicated the possibility of decentering distortion varying with focussing (see Table 2 and Figure 5, Brown, 1971). No

$$\Delta x_s = (1 - \frac{c}{s}) [P_1(r^2 + 2x^2) + 2P_2xy]$$

$$\Delta y_s = (1 - \frac{c}{s}) [P_2(r^2 + 2y^2) + 2P_1xy] \quad (10)$$

where s is the distance on which the lens is focussed. This result holds strictly only for points in the plane at s. As with symmetric radial distortion a further modification is required to account for the variability within the photographic field. This is accomplished through application of the scaling factor $\gamma_{ss'}$, developed in Brown (1971) and defined as

$$\gamma_{ss'} = \frac{s - c}{s' - c} \frac{s'}{s} \quad (11)$$

where s is, as above, the distance to the plane on which the camera is focussed and s' is the distance to the plane of the point under consideration. To generate the formula for decentering distortion for a point at distance s' when the camera is focussed at s one must merely mutliply the right hand sides of equations (10) by the factor $\gamma_{ss'}$

Experimental Confirmation.

All the lenses tested in early 1985 were found to have decentering distortion profiles (J_1r^2) which increase slowly in magnitude as the camera is moved further away from the plumb-lines, that is, J_1r^2 increased as Δc_s (focussing) decreased. An example is shown in Table 3 for a 120mm f-8 Nikkor-SW lens, measured at photographic scales of 1:8, 1:12, 1:16, and 1:20, respectively. The measured decentering profiles are shown in Figure 2.

TABLE 3. OBSERVED DECENTERING DISTORTION PROFILES AT r = 100 MM FOR VARIOUS PHOTOGRAPHIC SCALES AND CORRESPONDING PREDICTED VALUES FOR INFINITY FOCUS.

Camera: CRC-102	120-mm f/8 Nikkor-SW Lens	
Photo Scale	Value $J_{1s}r^2$ r = 100 mm	Predicted Values J_1r^2 at r = 100 mm (Infinity Focus)
1:8	28.9 μm	32.5 μm
1:12	29.6	32.1
1:16	29.7	31.6
1:20	31.0	32.5
	Mean Infinity Value = 32.2 μm	
	RMS = ±0.4	

The values of the decentering distortion profile for focus at infinity for a radial distance of 100mm were predicted from each of these calibrations using equation (7). Ideally all should produce the same result. The mean value of 32.2 micrometers has a standard error of only 0.4 micrometers. Similar values, averaging 0.5 micrometers were obtained from other camera calibrations. This not only provides good confirmation of the refined decentering model set forth in this paper but also serves as an indicator of the advances made by the combination of the CRC-1 and AutoSet.

Photogrammetric Implications.

The magnitude of the variation of decentering distortion is dependent on whether the adopted values of P_1 and P_2 are infinity values and on the difference in the distances s and s', i.e. the depth of field.

The fundamental correcting term $\gamma_{ss'}$ $(1 - \frac{c}{s})$ equals unity at infinity focus but for close range work can become quite significant. For example, if the profile of decentering distortion is 30 μm at a radial distance of 100 mm and P_1 and P_2 were calibrated at infinity, then at a photographic scale of 1:10 a 3 μm error will be made in the direction ϕ_0 of the maximum tangential distortion and up to 9 μm in the radial component of the decentering distortion in the direction $\phi_0 + 90^o$. A large variation in the depth of field for very close range work will exacerbate these errors.

While 30 micrometers of decentering distortion may seem large to users of aerial photogrammetric cameras, it is not uncommon for the types of lenses used in close range applications.

The term $\gamma_{ss'}$ $(1 - \frac{c}{s})$ is, of course, a scalar quantity. Consider the effect that this has on a bundle adjustment with self-calibration where P_1 and P_2 have been held invariant. If the same focal length setting had been used for all camera stations, assuming only one camera, then the P_1 and P_2 values returned from the self-calibration will be the best approximation for the decentering distortion at that focal length setting. The accuracy of the object coordinates will not have been degraded. If, on the other hand, several focal length settings were used, the results will be compromised since different decentering distortion values should have been computed for each focal length setting.

If the coefficient matrix for the decentering distortion terms P_1 and P_2 is modified by consideration of the term $\gamma_{ss'}$ $(1 - \frac{c}{s})$ the process of self-calibration will provide values of P_1 and P_2 corresponding to infinity focus. This technique is recommended.

Conclusions

Recent advances in technology have made the analytical plumb-line method a more powerful tool than ever before for lens calibrations. The development of the large format microprocessor-controlled CRC-1 camera and the sub-micrometer precision of the automated monocomparator AutoSet enabled determinations of the decentering distortion profile to be made to an accuracy sufficient to uncover a shortcoming in the conventional formulae.

Decentering distortion was found to be a function of camera focussing. The generalized form of the model is given by equations (10), modified when appropriate by the factor $\gamma_{ss'}$.

The new form of the model has been experimentally confirmed by a series of lens calibrations. Its adoption into algorithms for self-calibration is therefore recommended. The adoption of the infinity focus values as the "standard" for the terms P_1, P_2, J_1 and ϕ_0 is further recommended.

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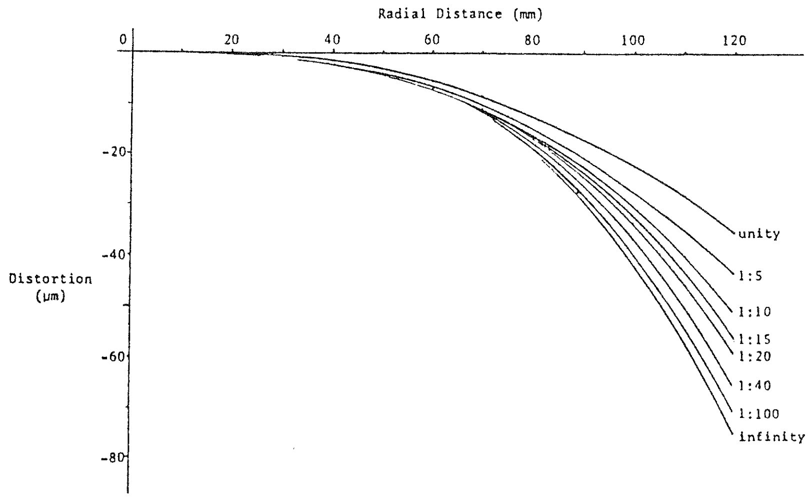

FIG.1. Gaussian radial distortion for varying photographic scales. Camera CRC-102, 240-mm *f*/9 Fujinon-AS lens. Values at 1:10, 1:15, and 1:20 determined from plumb-line calibration; other values computed from 1:10 and 1:20.

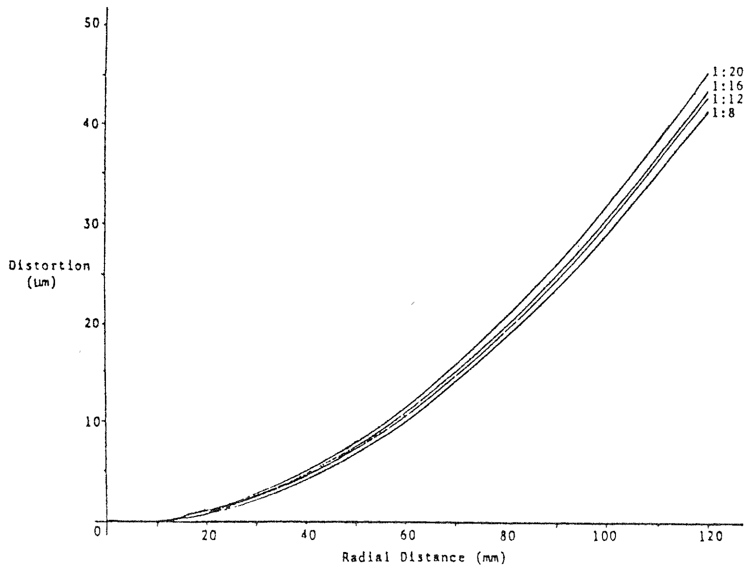

FIG.2. Observed decentering distortion profiles. Scales from 1:8 to 1:20, Camera CRC-102, 120-mm lens.