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Is the Request for Eight Fiducial Marks Justified?

The distance between the marks and the center of the photo is more important than the direction they are located in as viewed from the center as long as they are evenly distributed.

ABSTRACT: *This paper discusses methods for the correction of image deformation based on four and eight fiducial marks. It is demonstrated that eight fiducial marks—four located in the format corners, the other four in the middles of the format sides—do not yield an image deformation correction significantly better than that obtainable using only the four fiducial marks located in the middles of the format sides. A new eight-fiducial-mark arrangement has been proposed which yields a better image deformation correction especially for badly deformed photographs. However, none of the correction methods investigated leads to as an effective image deformation correction as a properly applied reseau correction.*

INTRODUCTION

A PRECISE METRIC evaluation of a photograph is only possible after several unknowns have been determined. Two of these unknowns are the coordinates of a center point, together with the calibrated focal distance and the lens distortion, needed to reconstruct the bundle of rays that formed the photographic image.

Such a center point can be derived from the frame of the photograph. However, a point so determined will not be very accurate because of the poor definition of the frame. Therefore, well defined fiducial marks have been added to cameras being used for taking photographs for surveying purposes.

In terrestrial cameras using glass plates, only two such marks were used in the beginning. These marks were located in the middle of the left and right sides of the rectangular format respectively and used to level the photograph and to determine the center point. Soon, two additional fiducial marks were added being located in the middle of the top and bottom sides of the format, mainly for the purpose of adjusting the horizontal parallax for stereo pairs taken with parallel optical axes.

The change-over from terrestrial to aerial photography for surveying purposes brought soon the advent of the quadratic format and of film, replacing the bulky glass plates. This new format then resulted in placing the fiducial marks into the frame corners. The definition of the fiducial marks was greatly improved because they were now also used to correct film deformation.

Finally, in recent years, eight fiducial marks have been requested in order to yield a better correction for film deformation, four of them located in the corners and the other four in the middles of the sides.

One should also note that, already in the last century, a reseau was introduced by astronomers to control image deformation. A reseau is a net of points (evenly distributed over the entire format) which is impressed upon the photographic image at the moment of exposure. A reseau being supported by a register glass plate is a rigid part of the camera. Therefore, the reseau crosses imaged on the photograph can be considered as a special type of fiducial marks.

In the following, the two primary functions of the fiducial marks in aerial photography will be discussed: their use for the determination of a center point of the photograph and for the correction of image deformation.

Determination of a Center Point

According to Roos[1], three different types of definitions for the interior orientation of a camera are available: mathematical, physical and technical definitions.

The mathematical definitions are very simple. A bundle of rays being straight lines originates from an object, meets in a projection center and continues intersecting the image plane. The bundle is centered around a projection axis. The image plane is perpendicular to the projection axis. The intersection point of the projection axis with the image plane is the principal point which at the same time is the geometrical center of the image.

Unfortunately, the physical definitions have to depart from these very convenient mathematical definitions. A bundle entering a lens does not intersect in one point. The bundle leaving the lens is not identical to that entering the lens. The plane of best definition is a curved, rotational symmetrical surface. The optical axis of the lens intersects the plane of best definition at the principal point.

The technical definitions have again to depart from the physical definitions because of the restricted technological abilities: a lens composed of different lenses will not have a unique optical axis, the plane of best definition will deviate from a rotational symmetrical surface, the plane of best definition will not be perpendicular to what emerges as overall optical axis. An image plane will have to be selected in such a way that the deviations from the plane of best definition become a minimum. A calibrated focal distance has to be derived so that the distortions, i.e., the deviations between the bundle of rays entering the lens and that emerging from the lens, are best distributed. According to the method of determination, several center points with a special meaning can be considered. In an ordinary camera having only fiducial marks along the camera frame, any of these center points will have to be referred to a center point derived from the fiducial marks.

The accuracy of a center point depends therefore on the determination of the reference system imaged on the photographs. Because the photograph is subjected to deformations, the reference center point derived from the fiducial marks will not be correct. Reseau photographs permit the investigation of displacement using reseau points having locations similar to the fiducial marks in ordinary cameras, and the center cross. These points are indicated in Figure 1.

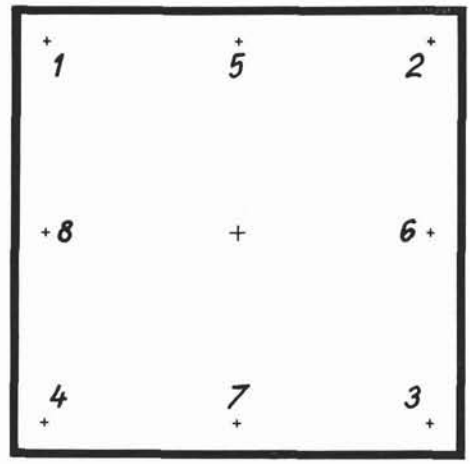

Fig. 1. Reseau points used in place of the eight fiducial marks, and center cross.

Using these 8 points in reseau photographs from 14 separate missions, the following values were derived:

- Mean values out of points 1 and 3, 2 and 4, 5 and 7, and 6 and 8 respectively;
- Mean values out of points 1 to 4, and 5 to 8 respectively;
- Mean value out of points 1 to 8;

and compared with the coordinates of the center cross.

The resulting differences are given in Table 1, first for reseau photographs from 14 separate missions, then for photographs from 11 missions, excluding the badly deformed photographs from 3 missions. The computations were applied after performing a linear conformal transformation using 8 fiducial marks for the determination of the transformation coefficients using Formulas 1 to 6 listed in the following section.

Interestingly enough, the smallest x displacement, i.e., in film direction, was obtained using two points only, 5 and 7, which are located on a line across the film direction. For the displacement across the film, i.e., in the y direction, the smallest value was obtained taking the mean out of the four points 5, 6, 7 and 8, i.e., the points located at the middles of the format sides. However, this

[1] W. Roos, "On the definition of fundamental concepts in photogrammetry," *Photogrammetria*, No. 3 1951–1952.

TABLE 1. UNCERTAINTIES IN THE DETERMINATION OF THE CENTER POINT OF A PHOTOGRAPH USING EIGHT FIDUCIAL MARKS

(14) Mean value derived from photographs from 14 missions.
(11) Mean value derived from photographs from 11 missions.

Points Used	*Values in Film Direction in 8m*		*Values Across the Film in 8m*	
	(14)	*(11)*	*(14)*	*(11)*
1, 3	39.5	8.9	41.4	18.3
2, 4	32.7	9.6	37.5	15.9
5, 7	5.4	5.8	12.3	8.8
6, 8	23.2	10.6	9.0	9.0
1, 2, 3, 4	35.3	6.7	11.1	10.7
5, 6, 7, 8	12.0	6.2	7.7	6.7
1 to 8	23.4	6.6	8.7	8.5

value is only slightly better than the one obtained using the two points 6 and 8 located on a line in the film direction. The smallest values derived from all the photographs are among those which did not change essentially by dropping the badly deformed photographs.

It might be concluded that the addition of four points located in the frame corners did not improve the result obtained using only the points located in the middle of the format sides. Further, a definition of the fiducial marks yielding a pointing accuracy of 5 μm is sufficient. Finally, the two center points derived by linear interpolation using either arrangement of four fiducial marks will in general not be identical because of image deformation.

CORRECTION OF THE IMAGE DEFORMATION

For an investigation of several methods for the correction of the image deformation the same photographs were used as in the previous chapter. All of these reseau photographs had a reseau spacing of 1 cm by 1 cm. Most of the photographs had 23 rows with 23 reseau points each, the other had only 21 rows with 21 points each.

The procedure for comparing the different correction methods will be explained first.

All the reseau points of a photograph were measured in a comparator. The readings were then corrected first for known deviations of the camera reseau from an ideal 1-cm-spaced grid, then for known comparator errors to ensure that any deviation from such an ideal grid could be attributed to image deformation. The next step consisted in a linear conformal transformation using all the reseau points for the determination of the transformation coefficients. Then 96 reseau points, having in most instances a spacing of 2 cm by 2 cm, were selected as *image points*. During the course of the investigation, the mean-square residuals derived from these 96 points were used to compare the different correction methods.

The mean-square residuals obtained after the initial linear conformal transformation varied between approximately 5 μm and 40 μm. To compare the relative improvements rather than the absolute values, the values obtained from any correction method were scaled by dividing them by the initial mean-square residual in position for the 96 points. Therefore, in Figures 2 to 6, the value 1.0 on the vertical axis indicates the mean square residual in position for the 96 *image points* after the initial linear conformal transformation using all the available reseau points. Then a mean value was derived from the scaled mean-square residuals. This value will in the following be referred to as *scaled mean value*.

In the investigation, the following *fiducial mark arrangements* were used:

I. Four corner fiducial marks, i.e. four réseau crosses positioned on the diagonals, as close to the corners as possible;
II. Four side middle fiducial marks, i.e. four réseau crosses located on the lines connecting the middles of the format sides, as close to the format edge as possible;
III. Eight fiducial marks, i.e. the arrangements I and II together.

Wherever possible, the following transformations were applied to these fiducial mark arrangements:

$$\begin{aligned} x &= a_0 + a_1x - b_1y \\ y &= b_0 + b_1x + a_1y \end{aligned} \tag{1}$$

$$\begin{aligned} x &= a_0 + a_1x + a_2y \\ y &= b_0 + b_1x + b_2y \end{aligned} \tag{2}$$

$$\begin{aligned} x &= (a_0 + a_1x + a_2y)/(1 + c_1x + c_2y) \\ y &= (b_0 + b_1x + b_2y)/(1 + c_1x + c_2y) \end{aligned} \tag{3}$$

$$\begin{aligned} x &= a_0 + a_1x + a_2y + a_4xy \\ y &= b_0 + b_1x + b_2y + b_4xy \end{aligned} \tag{4}$$

$$\begin{aligned} x &= a_0 + a_1x + a_2y + a_3x^2 + a_4xy + a_5y^2 \\ y &= b_0 + b_1x + b_2y + b_3x^2 + b_4xy + b_5y^2 \end{aligned} \tag{5}$$

$$\begin{aligned} x &= a_0 + a_1x + a_2y + a_3x^2 + a_4xy + a_5y^2 \\ &\quad + a_7x^2y + a_8xy^2 \\ y &= b_0 + b_1x + b_2y + b_3x^2 + b_4xy + b_5y^2 \\ &\quad + b_7x^2y + b_8xy^2. \end{aligned} \tag{6}$$

The results are shown in Figure 2.

The superiority of arrangement II over arrangement I is apparent. Within the frame of this investigation, arrangement II yields even better results than arrangement III, *if* the same transformations, 1, 2, or 3, are used. However, using additional degress of freedom in the form of third-order terms, a slightly better result has been obtained for arrangement III, i.e. the eight fiducial marks. The improvement using eight fiducial marks was better after exclusion of the badly deformed photographs.

The results of this initial investigation, i.e., in using the transformations listed above which include the transformations commonly used in connection with eight fiducial marks, do not meet the expectations concerning the use of four additional fiducial marks positioned in the frame corners.

In order to explain this phenomenon, an additional experiment was conducted. It consisted of four steps:

1. Four fiducial marks located either in the side middles or in the corners, were shifted towards the center of the photograph.
2. Eight fiducial marks were used; the four in the middles of the sides were maintained in their positions, the four in the corners were again shifted towards the center.
3. The eight fiducial marks were maintained in their positions and weights were attached to them.
4. The eight fiducial marks were moved to new locations and then again shifted towards the center.

STEP 1

The photographs used in this investigation were reseau photographs. Instead of the fiducial marks, reseau crosses in suitable positions were used. *Shifting of the fiducial marks* was achieved by simply using other reseau points. In this first step the fiducial marks were moved towards the center by selecting the nearest point to the reseau point used in the direction towards the center of the photograph. The results are shown in Figure 3.

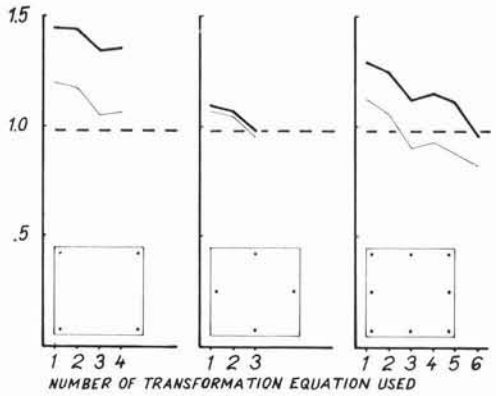

FIG. 2. Comparison of mean-square residuals in position for the four corner image marks, the four image marks located in the middle of the format sides, and for these eight image marks together.

All transformations used show, for the four fiducial marks located on the diagonals of the photograph, first a decline of the scaled mean values, then, starting at approximately 11 cm from the center, an increase upon moving the fiducial marks towards the center of the photograph. Using points close to the corners, significantly better results were obtained after exclusion of the badly deformed photographs.

For the side middle fiducial marks, the maximum distance to the center of the photograph amounts to 11 cm only. Analyzing the results obtained upon shifting these points towards the center, one might conclude that the same trend is evident as for the points located on the diagonals, for the same distance from the center. However, a value slightly worse than that for 10 cm distance to the center is here obtained for a distance of 11 cm. This can probably be explained by the fact that the deformations close to the edge of the film are somewhat different than in the remaining parts of photographs. This means that fiducial marks should *not* be po-

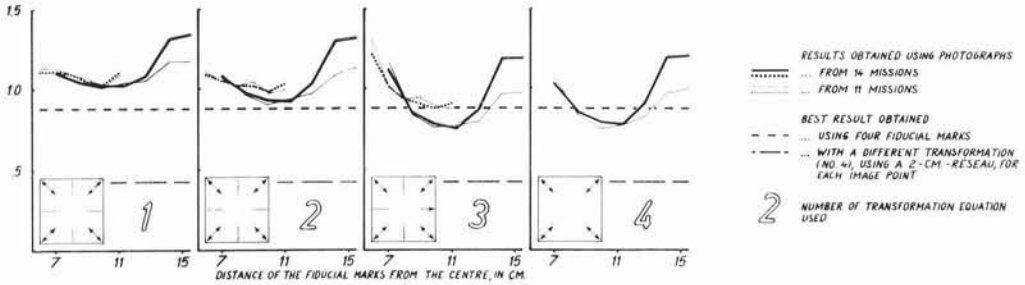

FIG. 3. Dependence of mean-square residuals in position from the distance between center and image marks, for the four corner marks and the four edge middle marks.

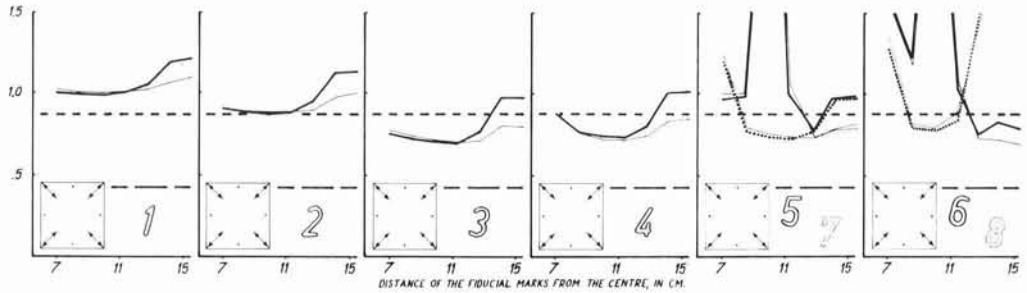

FIG. 4. Dependence of mean-square residuals in position from the distance between center and image marks, for eight marks. The corner marks are moved whereas the edge middle marks are maintained in their most favorable position.

sitioned *too close to the edge of the film and therefore also not too close to the edge of the format.*

STEP 2

The side middle fiducial marks were now maintained in their most favorable positions, i.e., 10 cm from the center. The points located on the diagonals were again moved towards the center of the photograph. The results are presented in Figure 4. The results for these eight fiducial marks when using transformations (1) to (4) were better than those obtained using only the points located on the diagonals, as could be expected. With the points located on the diagonals being not farther away from the center than approximately 13 cm, the results for eight fiducial marks were also better than the best results obtained taking only the four side middle fiducial marks. However, one must notice that this location is already within the picture frame and consequently fiducial marks in this location could not be part of the image frame.

The use of the transformations given in Equations 5 and 6 is most effective if the points located on the diagonals are located in the corners. These equations become even unsolvable if all the points are positioned on a circle with the center of the photograph as center point. If this is encountered, other transformations having the same number of coefficients can be used. Assuming four of the eight points are located on the axes of the image coordinate system, the following transformations can be used:

$$\begin{aligned} x &= a_0 + a_1x + a_2x^2 + a_5y + a_6yx + a_7yx^2 \\ y &= b_0 + b_1y + b_2y^2 + b_5x + b_6xy + b_7xy^2 \end{aligned} \tag{7}$$

$$\begin{aligned} x &= a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5y + a_6yx + a_7yx^2 \\ y &= b_0 + b_1y + b_2y^2 + b_3y^3 + b_4y^4 + b_5x + b_6xy + b_7xy^2. \end{aligned} \tag{8}$$

These two transformations did not lead to better results than those obtained using the projective transformation, Equation 3.

STEP 3

The results obtained so far have indicated that the results can be improved if the fiducial marks could be brought closer to the

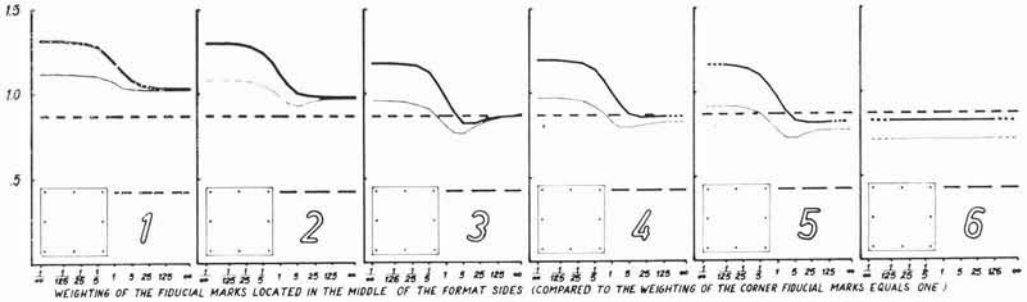

FIG. 5. Dependence of mean-square residuals in position from the weighting of the eight image marks. The weight of the corner marks always equals 1; the weight of the middle marks varies from 0 to infinity.

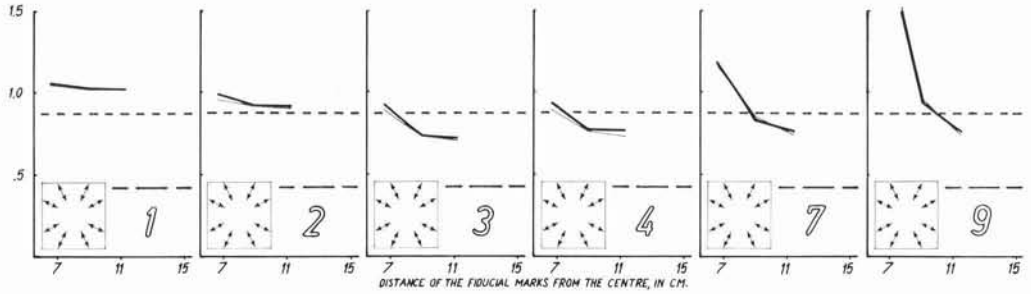

FIG. 6. Dependence of mean-square residuals in position from the distance between center and image marks, for eight marks equidistant from the center.

center. Corrections could also be attempted by giving the four points located in the middle of the sides a higher weight than the corner points.

During this investigation *weights* were introduced by using some points more often, namely twice, 5, 15, 44 and 132 times. Choosing a logarithmic scale, these numbers result in almost equal distances on the coordinate axes. The results are presented in Figure 5.

The values plotted to the very left are those obtained using the four corner fiducial marks only. Proceeding to the right, the weight of the four side middle fiducial marks increases. In the middle, the results for eight equally weighted fiducial marks are shown. To the very right the results for the four side fiducial marks only are plotted.

The results for eight weighted fiducial marks are approaching the two values obtained for four fiducial marks. The results from Equations 4 and 5 show the same tendency, although both transformations cannot be solved for four side middle fiducial marks. Equation 5 cannot be solved for four corner fiducial marks either. With Equation 6, i.e., with no redundant observations, the weights did not have any influence on the result.

For the four corner fiducial marks, there is a pronounced difference in the scaled mean value obtained including and excluding the badly deformed photographs. As their weight decreases in favor of the four side fiducial marks, the difference decreases also and becomes very small if the weight of the side fiducial marks is significantly higher than that of the corner fiducial marks.

Especially after the exclusion of the badly deformed photographs, the results are best if the weight of the side fiducial marks is five times as large as the weight of the corner fiducial marks.

STEP 4

Step 3 did not lead to a significant improvement compared to the results obtained for side middle fiducial marks only. Therefore, an entirely *new fiducial mark arrangement* was investigated.

The results obtained in Step 1 indicate that the distance between the fiducial marks and the center of the photograph is more important than the specific location of the marks along the frame of the photograph, within certain limits of course. Furthermore, the results of Step 1 can be interpreted as yielding 11 cm as the most favorable distance between the fiducial marks and the center of the photograph. However, the fiducial marks should not be placed too close to the edge of the format. Considering these two conclusions, eight reseau points 11.18 cm from the center, located 28.4° off the diagonals when viewed from the center, were selected. As these eight points are located on a circle, Transformations 5 and 6 cannot be used. Furthermore, Transformation 8 cannot be solved; hence, it was modified slightly:

$$\begin{aligned} x &= a_0 + a_1x + a_2x^2 + a_3x^3 + a_5y + a_6yx \\ &\quad + a_7yx^2 + a_8yx^3 \\ y &= b_0 + b_1y + b_2y^2 + b_3y^3 + b_5x + b_6xy \\ &\quad + b_7xy^2 + b_8xy^3. \end{aligned} \tag{9}$$

The results are shown in Figure 6. They are the best results obtained in this investigation using fiducial marks that could be part of the image frame. They are almost identical to the results obtained for eight fiducial marks located 11 cm from the center in the side middles and on the diagonals. However, as pointed out earlier, the latter fiducial mark arrangement cannot be used in practice. The similarity of these results seems to indicate once more that *the location of the fiducial marks is not as important as*

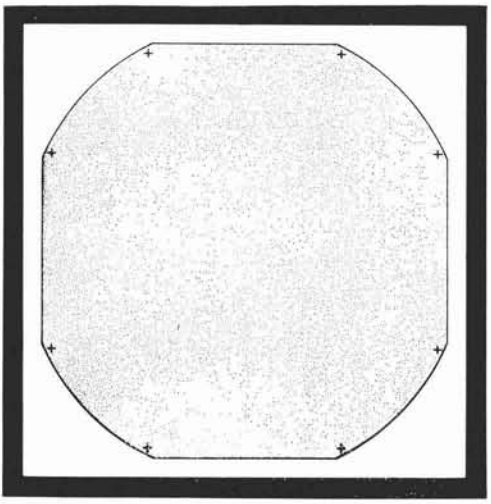

FIG. 7. Shaded area denotes recommended area for fiducial mark location.

their distance from the center of the photograph.

One more observation deserves attention. In Figure 6 there is hardly any difference in the scaled mean values after exclusion of the badly deformed photographs. The fact that the effectiveness of the correction appears to be equal for normally and for badly deformed photographs cannot be stressed enough because the more deformed the photographs are, the more important the image deformation corrections become. The fiducial marks should fall within the shaded area of Figure 7.

So far, it has always been assumed that image points are evenly distributed over the entire image. At least during the aerial triangulation, this is not true. The points used are located near the image corners, aside from one point or point group close to the center of the photograph. Assuming standard overlaps of 60 percent in longitudinal and 20 percent in lateral direction and a format of 23 cm by 23 cm, these points are a distance of approximately 2.3 cm from the image frame. In Figures 8 to 10, the image deformation of three different reseau photographs is shown. The positions of the nine orientation points as well as the positions of eight fiducial marks as used now are also shown. Although the distance between the corner fiducial marks and the nearest orientation points amounts to only approximately 2 cm, distinct differences in the vectors at these points can be observed; this is not so of fiducial marks at the middle of the format side. It might be added that the orientation points near the format

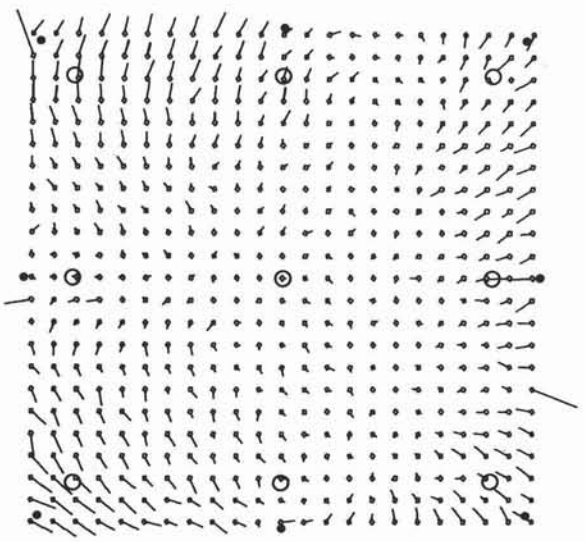

FIG. 8. Aerial photograph normally deformed. Local deviations were probably caused by isolated cases of poor film flattening. Dots indicate the positions of the eight fiducial marks, as presently used, circles the approximate positions of the orientation points.

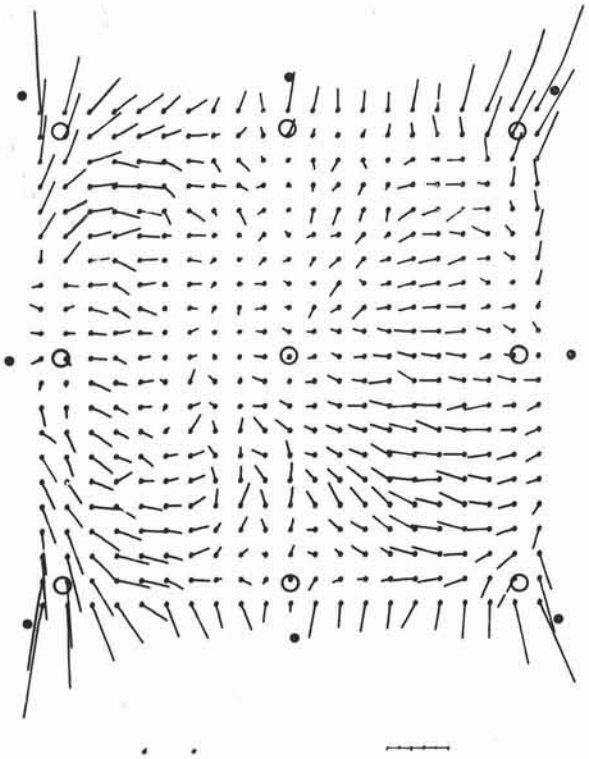

FIG. 9. Aerial photograph with abnormal deformation probably caused by an incorrect film drying technique. Dots and circles have the same meaning as in Figure 8.

corners are just outside the shaded area of Figure 7 within which the fiducial marks should be positioned.

SUMMARY AND CONCLUSIONS

Throughout the investigation it became apparent that the distance between the fiducial marks and the center of the photograph is more important than the direction they are located in as viewed from the center of the photograph as long as the fiducial marks are evenly distributed.

Because of that, four fiducial marks located in the middle of the format sides can be expected to yield a more effective image deformation correction than eight equally weighted fiducial marks, four of them being located in the middle of the sides, the other four in the frame corners, if conventional transformation formulas are used and if points evenly distributed over the entire image are considered.

To facilitate comparison of the different methods discussed in this paper, two straight lines have been added to Figures 3 to 6, indicating the best value obtained using only four side fiducial marks (0.87), and the value obtained using an individual image point correction based on a 2 cm reseau (0.43). These figures reveal that using eight fiducial marks located in the frame corners and in the middles of the format sides offered, with either proper weighting or with the application of a transformation having more degrees of freedom than those usable with only four fiducial marks, slightly better results than those obtained for four side fiducial marks. A still better result was obtaind using an entirely new fiducial mark arrangement which takes advantage of the most favorable distance between the fiducial marks and the center of the photograph, this distance being approximately 11 cm for the format of 23 cm by 23 cm. This new arrangement has the additional advantage of yielding an equally effective image deformation correction for normally and badly deformed photographs.

Figures 3 to 6 also reveal that in this investigation none of the methods which were based on eight fiducial marks yielded an image deformation correction as effective as a proper reseau correction. The restriction in effectiveness is a result of the fact that *fidu-*

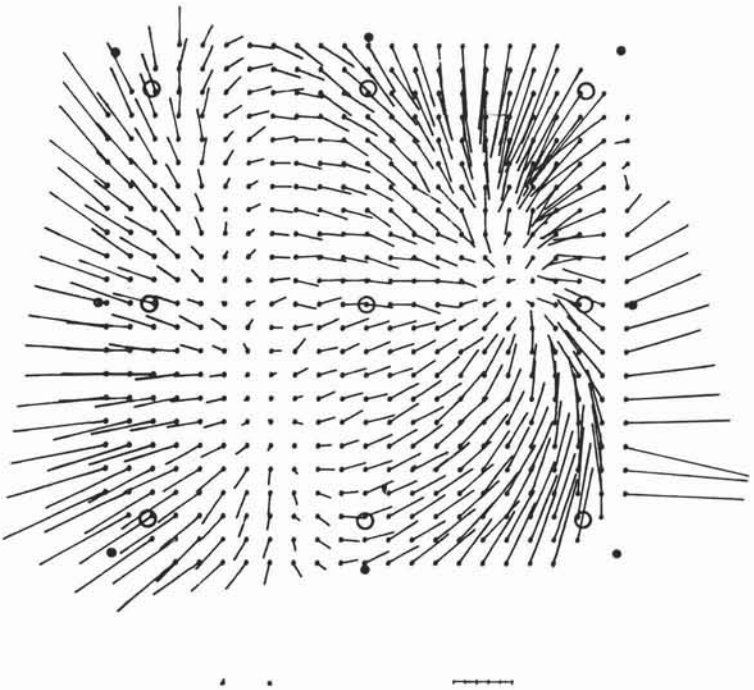

FIG. 10. Aerial photograph with abnormal deformation probably caused by improper film flattening—note the right hand side. Dots and circles have the same meaning as in Figures 8 and 9.

cial marks located along the frame are not able to account for any local deformations within the photograph.

Other methods could be considered, especially methods based on an individual transformation of each point. However, it is doubtful whether such procedures could yield a significantly better image deformation correction than the methods described.

Finally, the fiducial marks should not be located too close to the format edge and hence, to the film edge. Further, the projective transformation, Equation 3, performed generally best.

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