ESTIMATING ACCURACY OF PHOTOGRAMMETRIC DATA—MECHANISM AND IMPLEMENTATION

By Sagi Filin[1] and Yerahmiel Doytsher[2]

ABSTRACT: A method for estimating the accuracy of data collected from aerial photographs is presented. The emphasis is on its implementation in accuracy estimation when gathering data for digital terrain model databases. The method, based on error propagation of the photogrammetric solution process, facilitates the determination of accuracy for applications based both on direct photogrammetric measurements and on digital terrain model databases in addition to determining the measurements' accuracy. The mechanism and implementation at different levels of complexity is described, and various aspects of the proposed method are demonstrated, especially with applications dealing with the altimetric component of the terrain.

INTRODUCTION

Although GIS (geographic information system) databases cover and handle many aspects of accurate representation, accuracy estimation is usually not treated in terms of data management, analysis, and planning. At best, accuracy is estimated by way of absolute orientation SD (Standard Deviation).

Attempts to evaluate accuracy of data derived from aerial photographs and from digital terrain model (DTM) layers are usually referred to in accuracy evaluation as an empirical procedure, i.e., using statistical tests in order to evaluate accuracy (Acharya and Chaturvedi 1997). A project conducted by Working Group 3 of Commission 3 of the ISPRS (Torlegard et al. 1986) applied comparative tests to 2,500 points collected from stereo-models of different scales in order to evaluate the data and DTM accuracy. Other attempts (Ley 1986) and (Li 1991) focused on determining an optimal data set in order to estimate the accuracy of a DTM layer. It seems, however, that by employing these methods an indirect strategy for accuracy estimation was chosen, instead of referring to the actual problem and analyzing the factors affecting the accuracy estimate of the ground coordinates. An attempt to refer to the photogrammetric process was introduced by Blace (1987), who suggested a summation of the aerial-triangulation SD, the absolute orientation SD, and the sampling SD.

This paper describes a direct approach for evaluating the accuracy estimate for data based on aerial photographs. The described method is constructed in a general form that can fit any photogrammetric model.

ERROR PROPAGATION IN PHOTOGRAMMETRIC PROCESS

Photogrammetric orientation is the basic step for coordinate transformation from the image reference coordinate system to the ground system. Its prin-

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[1]Prof., Dept. of Civ. Engrg. (Geodesy), Technion–Israel Inst. of Technol., Haifa 32000, Israel. (Currently a PhD Student at Ohio State Univ.)

[2]Prof., Dept. of Civ. Engrg. (Geodesy), Technion–Israel Inst. of Technol., Haifa, Israel.

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cipal concept is the determination of the position and rotation angles of the perspective center in relation to the ground coordinate system. However, different cameras models—varying from the calibrated frame camera to panoramic photographs, SPOT images, etc.—prevent the determination of a universal model. (For instance, a SPOT-image orientation should include dynamic parameters, such as linear and angular velocities.) In addition, an orientation of a stereo-pair (the state in which GIS data are usually collected) can be determined both by: using a two step procedure—relative orientation and absolute orientation—or by separately computing an exterior orientation for each photograph. Therefore, when discussing the error propagation, the camera model and the orientation method should be defined.

The paper demonstrates the error propagation mechanism applied to the commonly used stereo-pair frame photograph transformation model. This orientation model is based on the two steps of relative and absolute orientation. This commonly used model is considered to be complicated, since the error propagation involves a transformation through two object spaces.

Relative Orientation

The relative orientation is the determination of the relative spatial position between two adjacent photographs, making each homologous pair of rays intersect in space. It is expressed by the coplanarity condition [Manual of Photogrammetry (1980); Moffit and Mikhail (1980)] and is based on $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = 0$, which is satisfied by the determinant

$$\begin{vmatrix} P_x & P_y & P_z \\ u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \end{vmatrix} = 0 \qquad (1)$$

where the subscripts denote the image from which the measurements were taken, and

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{R}(\kappa, \varphi, \omega) \begin{bmatrix} x \\ y \\ f \end{bmatrix}$$

where x, y = photograph coordinates; f = focal length; $\mathbf{P}$ = vector crossing through the two perspective centers; and $\mathbf{R}(\kappa, \varphi, \omega)$ = rotation matrix, formed by the three rotation angles.

Computation of the transformation parameters using the least squares adjustment (Mikhail 1976) enables evaluation of their accuracy estimate. The variance-covariance matrix, (2) presents them:

$$\sum\nolimits_{rr} = \sigma_0^2 * N^{-1} \qquad (2)$$

where the subscript rr in Σ_{rr} denotes the relative orientation, and

$$N^{-1} = \mathbf{A}^T * A, \quad \sigma_0^2 = \frac{\mathbf{V}^T * \mathbf{V}}{n - u}$$

where $\mathbf{A}$ = design matrix; $\mathbf{V}$ = residuals vector; n = number of observations; and u = number of unknowns.

Stereo-Model Reference Coordinate System

By generating the above spatial relation, a 3-D-reference coordinate system is generated. Coordinates are computed by first applying the transformation parameters to the photograph coordinates and then applying the collinear rule. By using rotation angles as transformation parameters, model coordinates are computed by

$$Z_m = b * \frac{w' * w''}{w'' * u' - w' * u''}$$

$$X_m = Z_m * \frac{u'}{w'}$$

$$Y_m = \frac{Z_m}{2} * \left(\frac{v'}{w'} + \frac{v''}{w''}\right) \tag{3}$$

Due to the SD of the relative orientation transformation parameters, the model coordinates will inherit errors. These are evaluated by using error propagation, where the variance-covariance matrix is defined as

$$\sum_{mm} = \mathbf{F}_m * \sum_{rr} * \mathbf{F}_m^T \tag{4}$$

The **F** matrix components are computed by a derivation of (3) using the chain rule, as demonstrated by the following:

$$\frac{\partial X_m}{\partial \kappa} = \frac{\partial X_m}{\partial u} * \frac{\partial u}{\partial \kappa} \cdots$$

$$\frac{\partial Y_m}{\partial \kappa} = \frac{\partial Y_m}{\partial u} * \frac{\partial u}{\partial \kappa} \cdots$$

$$\frac{\partial Z_m}{\partial \kappa} = \frac{\partial Z_m}{\partial u} * \frac{\partial u}{\partial \kappa} \cdots \tag{5}$$

Absolute Orientation

When a 3-D coordinate system is formed, an absolute orientation (a projective transformation including scaling, rotation, and translation between the model and ground coordinates) can be formed. The mathematical model is given by

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \lambda * \mathbf{R}(\kappa, \varphi, \omega) * \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} \tag{6}$$

where X, Y, Z = ground coordinates; x, y, z = model coordinates; X_0, Y_0, Z_0 = translation parameters between the two coordinate systems; and λ = scale between the two coordinate systems.

Transformation parameters are usually solved using an adjustment by means of observation equations. However, observation equations disregard some of the observations' accuracy estimates. Ignoring this component may affect the accuracy of the solution.

The absolute orientation model is composed of two sets of observations —the model coordinates and the ground control points. The *ground control points* accuracy estimate is determined by the measuring technique. In surveying (conventional or global positioning system), coordinates accuracy is a function of the position determination, while the accuracy of photogrammetric controlled pass points is determined by the aerial-triangulation SD.

The *model coordinates* accuracy estimate is evaluated as just described. In both cases, however, the term accuracy should refer to the variances and the covariances (if known, especially for the ground control points).

Since both the source and target reference coordinate systems contain errors, adjustment by the observation equation model becomes inadequate. A satisfying model that enables an inclusion of more than one observation per equation is the adjustment-by-observations-and-conditions model, which is expressed by

$$\mathbf{A}*X + \mathbf{B}*\mathbf{V} - \mathbf{W} = 0 \tag{7}$$

And solved by

$$\mathbf{A}^T*(\mathbf{B}*\mathbf{P}^{-1}*\mathbf{B}^T)^{-1}*\mathbf{A}*\mathbf{X} + \mathbf{A}^T*(\mathbf{B}*\mathbf{P}^{-1}*\mathbf{B}^T)^{-1}*\mathbf{W} = 0 \tag{8}$$

where $\mathbf{A}$ = observations matrix; $\mathbf{V}$ = residuals vector; $\mathbf{B}$ = conditions matrix; $\mathbf{P}$ = weight matrix; $\mathbf{X}$ = vector of unknowns; and $\mathbf{W}$ = vector of disclosures. A transformation model that agrees with this adjustment model is expressed by

$$\begin{bmatrix} X_g + V_{x_g} \\ Y_g + V_{y_g} \\ Z_g + V_{z_g} \end{bmatrix} = \lambda*\mathbf{R}(\kappa \quad \varphi \quad \omega)*\begin{bmatrix} X_m + V_{x_m} \\ Y_m + V_{y_m} \\ Z_m + V_{z_m} \end{bmatrix} + \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} \tag{9}$$

where the subscripts g and m denote the coordinate source, ground and model, respectively.

Two elements are considered as supplements to the adjustment-by-observation equation model, the condition ($\mathbf{B}$) and weight ($\mathbf{P}$) matrices. $\mathbf{P}$ is composed of the two variance-covariance matrices, the ground control points matrix and the model matrix. $\mathbf{B}$ and $\mathbf{P}$ are defined as

$$\mathbf{B} = \begin{bmatrix} \begin{pmatrix} \lambda*\mathbf{R}_{3\times3} & 0 \\ 0 & \ddots \end{pmatrix} & 0 \\ 0 & (I_{3n\times3n}) \end{bmatrix} * \begin{bmatrix} Vx_{m1} \\ Vy_{m1} \\ Vz_{m1} \\ \vdots \\ Vx_{g1} \\ Vy_{g1} \\ Vz_{g1} \\ \vdots \end{bmatrix} \tag{10}$$

$$\mathbf{P} = \begin{bmatrix} \sum_{3n*3n} \text{model} & 0 \\ 0 & \sum_{3n*3n} \text{ground} \end{bmatrix}^{-1} \tag{11}$$

The variance-covariance matrix ($\Sigma_{aa} = \sigma_0^2*N^{-1}$) components are computed by

$$N = \mathbf{A}^T(\mathbf{B}*\mathbf{P}^{-1}*\mathbf{B}^T)*\mathbf{A}, \quad \sigma_0^2 = \frac{\mathbf{V}^T*\mathbf{P}*\mathbf{V}}{n - u}$$

Variance and Covariance Ground Coordinates

Ground coordinates measurement can be considered as an implementation over the absolute-orientation transformation. Therefore their accuracy estimate is affected by the absolute-orientation parameters SD. This accuracy estimate is evaluated by using error propagation. Two elements affect the coordinates accuracy estimate: the absolute orientation transformation SD and the model coordinates SD. The variance-covariance propagation is applied to the projective transformation.

Variance-covariance propagation expressing those two factors is defined as

$$\sum_{g-crd} = \mathbf{F}_1 * \sum_{aa} * \mathbf{F}_1^T + \mathbf{F}_2 * \sum_{mm} * \mathbf{F}_2^T \tag{12}$$

where Σ_{g-crd} = ground coordinates covariance matrix; Σ_{aa} = absolute orientation covariance matrix; Σ_{mm} = model coordinates covariance matrix; $\mathbf{F}_1$ = elements referring to the absolute orientation parameters; and $\mathbf{F}_2$ = elements referring to the model coordinates.

$\mathbf{F}_1$ is derived from the projective transformation model as described above, and $\mathbf{F}_2$ is derived from $\partial L_{\text{model}}/\partial L_{\text{ground}}$ and will have the form of $\boldsymbol{\lambda} * \mathbf{R}$.

One other component should be included in the variance-covariance matrix, the sampling accuracy estimate. Some optional methods are valid for its evaluation. Torlegard et al. (1986) evaluated the sampling accuracy as 0.2–0.4‰ (per-mill) of the flight height for flat terrain and 1–2‰ for mountainous terrain. Blace (1987) suggests using s_{setup} as the accuracy estimate. It can also be evaluated simply by using statistical tests referring to a specific operator. No matter what method is adopted, the sampling accuracy estimate should be added as a variance to the altimetric variances.

Evaluation of Mechanism

Some questions are raised with respect to the described mechanism, such as what the contribution of its components might be, and what the necessities. The more important questions ones refer to the impact of the accuracy estimate of the ground control points, the effect of the accuracy estimate of the measurements on the photogrammetric solution, and the actual contribution of the accuracy estimate of the model coordinates. The following subsections attempt to answer these questions.

The evaluation mechanism is presented via a 1:70,000-scale stereo-model based on pass points with a ±5 m accuracy estimate. Its absolute orientation, based on adjustment by observation equations, has produced an SD of only ±2 m. This low value represents the residuals within an internal coordinate system created by the photogrammetric block formation. In order to check the effect of the accuracy estimate inclusion, several accuracy estimate values for the ground controls were used. Among them were ±1 m (an accuracy estimate close to the actual residuals) and ±5 m (the actual accuracy estimate).

Model Coordinate SD Contribution

The effect of the model coordinates' SD is discussed twice; first while evaluating their effect on the absolute orientation, and second when the accuracy estimate of a ground coordinate is discussed. Elimination of this component will simplify and accelerate the accuracy estimate computation. First their effect on the absolute orientation is discussed.

Evaluating the contribution of the model coordinates component showed a nonnegligible value of about ±1.5 m for a 1:70,000-scale stereo-model. The computation is, however, affected not only by the model coordinates' accuracy but also by the ground coordinates' accuracy. Both effects are embedded within the $\mathbf{B} * \mathbf{P}^{-1} * \mathbf{B}^T$ matrix, but for the purpose of the analysis it is reduced to only variance summation $\sigma_1^2 + \sigma_{ground}^2$ (where σ_1 is the model coordinates' SD in ground values).

When an accuracy estimate of ±1 m was used for the ground coordinates, the model coordinates' SD contributed a significant value to the gross accuracy estimate. This is not only due to the variance summation but also due to the covariance rates. However, when accuracy estimates of ±3 m or larger were used, the ground control accuracy became dominant and the model coordinates' contribution turned to be negligible. This is explained by variances summation.

Effect of Accuracy Estimate on Photogrammetric Solution

The term "photogrammetric solution" relates to the transformation parameters, the photogrammetric solution SD, the residuals, and the variance-covariance matrix. In order to evaluate the accuracy estimate's effect on the photogrammetric solution, three cases were considered: adjustment without accuracy estimate, adjustment by using an accuracy estimate of ±1 m for the ground control points, and adjustment by using ±5 m ground control points. As presented in (11), the accuracy estimate affects the adjustment via the weight matrix.

Although different weight matrices were used, the variation in the transformation parameters was minor (Table 1). These changes can be explained by the function and effect of the weight matrices—a determination of the significance of each observation equation with respect to the others, and with only minor effect on the geometric construction. Because of these results the variation between the residuals will be minor as well. The accuracy estimate effect on the SD and variance-covariance matrices, however, was much more dominant (Table 2). Since the variance-covariance matrix parameters as derived from (12) effect the ground control point SD proportionally, this effect becomes significant.

The results show that while the adjustment without accuracy estimation produced optimistic results, and probably corresponds well to the photogrammetric block, only the inclusion of the ±5 m accuracy estimate produced more realistic values regarding the actual ground control accuracy. Thus the inclusion of accuracy estimation of the ground control points when comput-

TABLE 1. Transformation Parameters for Computation with Different Weight Matrices

Accuracy estimation (1)	λ (2)	X_0 (m) (3)	Y_0 (m) (4)	Z_0 (m) (5)	κ (rad) (6)	ϕ (rad) (7)	ω (rad) (8)
None	55.71990	5,053.20	7,607.77	10,339.54	−1.61797	0.00947	−0.05900
±1 m	55.71958	5,051.92	7,607.87	10,339.43	−1.61791	0.00937	−0.05902
±5 m	55.71990	5,053.14	7,607.78	10,339.53	−1.61796	0.00947	−0.05901

TABLE 2. Transformation Parameters SD

Accuracy estimation (1)	σ_0^2 (m) (2)	λ (3)	X_0 (m) (4)	Y_0 (m) (5)	Z_0 (m) (6)
None	4.119	0.00007	3.5029	10.1565	3.7072
±1 m	3.911	0.00032	6.1644	10.7894	3.8764
±5 m	1.000	0.00050	22.0634	61.9325	22.5913

ing an absolute orientation is recommended, especially when the control points are pass points. Although it seems that inclusion of the ±1 m accuracy estimate did not have any effect, the weight matrix effect becomes more evident when analyzing the correlation matrices derived from the variance-covariance matrices.

Ground Coordinates Accuracy Estimate

An analysis of the mechanism's effect on the ground coordinates accuracy estimate requires a discussion over two issues. The first concerns the affect of the measurements' accuracy estimation and the second concerns the actual necessity of employing the entire formulation described above instead of using a fixed value (usually the absolute orientation SD).

Experiments performed to evaluate the effects of including the measurement SD are presented in Table 3. The first row presents the accuracy estimate derived from an adjustment where no weight matrix was employed and by using only the effect of absolute orientation. The second row presents the same case but with the inclusion of the model's SD contribution. The third row presents the results of computation with a weight matrix while using only the absolute orientation effect; the fourth row presents a complete computation.

It can be seen that ignoring the accuracy estimate leads to an unrealistic accuracy estimate for ground coordinates, while its inclusion has succeeded in determining a realistic accuracy estimate for the computed ground coordinates. The difference between the values at rows 3 and 4, caused by including and omitting of the model coordinates SD, should be mentioned; here their effect is more significant than the effect they had on the absolute orientation adjustment.

In order to evaluate the rate of change of the accuracy estimate within the stereo-model, three dispersed points were checked (Table 4). The first and third were located on opposite edges of the model and the second was located at the center. The accuracy estimate varies from ±2.2 m in the center to ±3.9 m at the edges, a variation of 1.5 m. A presentation of the accuracy estimate

TABLE 3. Accuracy Estimation Results (in Meters) for Different Computations

Accuracy estimation absolute/relative	Point #1			Point #2		
	Vx	*Vy*	*Vz*	*Vx*	*Vy*	*Vz*
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Absolute—unweighted	1.30	1.30	1.48	0.83	0.83	0.83
Absolute and Relative—unweighted	1.61	1.58	2.14	0.89	1.00	1.61
Absolute—±5 m weighted	3.21	3.21	3.62	2.05	2.05	2.07
Absolute & Relative—±5 m weighted	3.46	3.47	4.25	2.09	2.14	2.88

TABLE 4. Accuracy Estimate (in Square Meters) of Three Dispersed Points within Stereo-Model

—	σ_x	σ_y	σ_z
(1)	(2)	(3)	(4)
Point #1	12.0	11.9	18.1
Point #2	4.4	4.7	8.3
Point #3	12.1	11.7	15.5

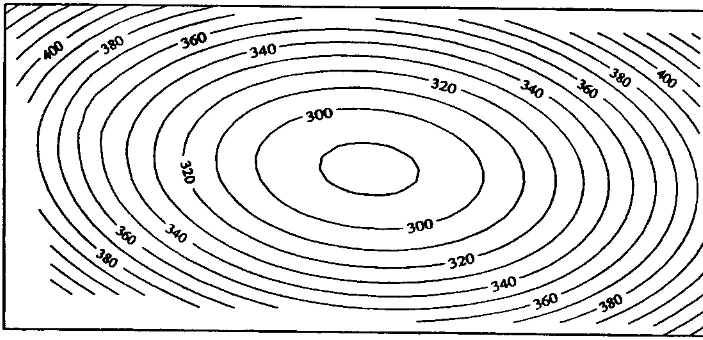

FIG. 1. Equal-Error Contours of Photogrammetric Model

variation throughout the model can be graphically formed by an equal-error contours (contours based on the SD values) generation.

Based on an accuracy estimate of ±5 m, Fig. 1 depicts the typical form of the contours (errors depicted in centimeters), their variation, and their magnitude at any given point. It seems that the rate of change from the model's center to its edges, which varies from ±3 m at the center to ±4.4 m at the edges (a nonnegligible difference of 1.4 m), points out that no fixed value will adequately present the accuracy estimate of the ground coordinates. Tests made on smaller-scale models with accurate ground control points have shown that, although the accuracy estimate magnitude was smaller, the rate of change (proportional) could not be ignored.

Covariance between Measurements

Relations between measurements, known as covariances, have not been presented up to this stage. Although they are insignificant with respect to a single measurement or separate measurements, their significance is revealed when several measurements are involved in a computation. Table 5 presents the variance-covariance matrix of three consecutive points (in a 2-km-distance step).

Analysis of the covariance ratios, regarding the "Z" component, shows a 97% correlation between the first two points and an 89% correlation between the first and third points (a distance of 4 km). Analysis of the covariance rate of change as a function of the distance (Table 6) depicts nonnegligible covariance values at distances that logically would not be related to each other. Two points, 8 km apart, share a covariance that represents a 50% correlation.

TABLE 5. Variance-Covariance Matrix for Three Points

— (1)	Point #1 (2)			Point #2 (3)			Point #3 (4)		
Point #1	**9.1**	0.3	2.1	**7.3**	0.1	2.1	**5.4**	−0.0	2.1
	0.3	**9.5**	1.0	0.2	**7.6**	1.0	0.2	**5.7**	1.0
	2.1	1.0	**13.6**	1.2	0.8	**11.6**	0.4	0.6	**9.7**
Point #2	7.3	0.2	1.2	**6.1**	0.1	1.2	**5.0**	0.0	1.2
	0.1	7.6	0.8	0.1	**6.5**	0.8	0.1	**5.3**	0.8
	2.1	1.0	11.6	1.2	0.8	**10.5**	0.4	0.6	**9.3**
Point #3	5.4	0.2	0.4	5.0	0.1	0.4	**4.5**	0.0	0.4
	−0.0	5.7	0.6	0.0	5.3	0.6	0.0	**4.9**	0.6
	2.1	1.0	9.7	1.2	0.8	9.3	0.4	0.6	**8.8**

TABLE 6. Variance-Covariance (2 km Steps)

— (1)	0 km (2)	2 km (3)	4 km (4)	6 km (5)	8 km (6)	12 km (7)
0 km	**13.63**	11.71	9.73	7.74	5.73	2.07
2 km	11.71	**10.53**	9.31	8.07	6.82	4.29
4 km	9.73	9.31	**8.83**	8.35	7.86	6.46
6 km	7.74	8.07	8.35	**8.63**	8.89	8.63
8 km	5.73	6.82	7.86	8.89	**9.90**	10.78
12 km	2.07	4.29	6.46	8.63	10.78	**15.47**

Since many computations use adjacent observations, covariance relevancy increases. The distances between measurements in common cases—i.e., elevation extraction from DTM databases—are usually very short (usually around fifty or a hundred meters). The correlation between measurements in such cases approaches nearly 100%.

ACCURACY ESTIMATE IMPLEMENTATION

Accuracy estimation can be divided into several levels of complexity when implemented on direct measurement, on derived applications, or on DTM databases and applications. Accuracy estimation on direct measurements is, as stated, the simplest of all, and all that is needed in such cases is to use (12) to extract the relevant variance, usually the altimetric values. However, most applications, being based on measured data, involve several measurements related by computation. Such cases mandate taking into account the interrelationship between the measurements. The accuracy estimate evaluation of the computation result will be calculated by determining the error propagation ($\Sigma_c = \mathbf{F} * \Sigma_{\text{coords}} * \mathbf{F}^T$) as a function of the computation formula. For a given function [$\mathbf{C} = f(x_1, y_1, h_1, \ldots x_n, y_n, z_n)$], $\mathbf{F}$ is determined by

$$\mathbf{F} = \left[\frac{\partial f}{\partial x_1}\,\frac{\partial f}{\partial y_1}\,\frac{\partial f}{\partial z_1}\cdots\frac{\partial f}{\partial x_n}\,\frac{\partial f}{\partial y_n}\,\frac{\partial f}{\partial z_n}\right] \quad (13)$$

The following sections present the implementation of, and discuss aspects of, the accuracy estimate effect. Implementation on direct measurements is presented using the profile measurement application.

Profiles

A profile is defined as a set of measurements along a line, usually obtained by direct measurement or computed by DTM interpolation. The profile discussed here was generated by direct measurement. An accuracy estimate for a profile is essential for evaluating derived applications, such as evaluating the volume SD, evaluating the visibility determination SD, etc.

Profile accuracy is evaluated by determining the variances of the altimetric component. The variance for each data point is extracted from its variance-covariance matrix. Its graphical representation is depicted in Fig. 2 (the profile is represented by a thick line while the SD buffer is represented by thin lines).

Accuracy Estimate for Calculated Distance

A calculated planimetric distance is a simple and commonly used function for the purpose of distance evaluation. The distance is defined by the well-known equation

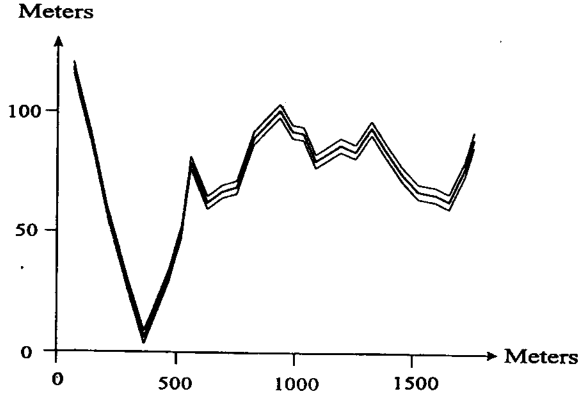

FIG. 2. Profile SD

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Its SD is usually evaluated by using a variance-based error propagation, and by assuming an equal precision in coordinate axes with no correlation: $m_{x1}^2 = m_{y1}^2 = m_{x2}^2 = m_{y2}^2 = m^2$ (where m denotes the accuracy estimate of the coordinates). The SD is given by $m_D^2 = 2*m^2$.

By using the matrix form of the error propagation (where the variance-covariance matrix is formed by extracting the planimetric variances and the relevant covariances for each point), the SD is expressed as follows:

$$m_D^2 = \mathbf{F} * \sum\nolimits_{p_1,p_2} * \mathbf{F}^T \quad (14)$$

where Σ_{p_1,p_2} is the variance-covariance matrix of the two points, and

$$\mathbf{F} = \left[\frac{dx}{D} \; \frac{dy}{D} \; \frac{-dx}{D} \; \frac{-dy}{D}\right]$$

An accuracy estimate evaluated by variance only, produced a result twice as large (i.e., half as accurate) as that evaluated by including the covariance. In addition, the variance-based method is insensitive to the variation in the distance between the measured points. The evaluated SD remains the same for long distances as well as for short ones, notwithstanding the covariance variation (as demonstrated previously).

Accuracy Estimate for Average Elevation

Computing an average elevation between two points provides another example of the evaluation technique and the effect of covariance on the evaluated SD. The average elevation is defined as: $h_m = (h_1 + h_2)/2$.

An SD evaluation based on error propagation by variance only (using the assumption $m_{h_1}^2 = m_{h_2}^2 = m^2$) will lead to the following result: $m_{h_m}^2 = m^2/2$.

By using the error propagation matrix form (based on the same assumption described above, and assuming covariance is 0.9 times the variance—a common ratio as depicted in Table 6), the average height SD is evaluated as follows:

$$(0.5 \quad 0.5)\begin{pmatrix} m_h^2 & 0.9m_h^2 \\ 0.9m_h^2 & m_h^2 \end{pmatrix}\begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} = 0.95m_h^2$$

A comparison between the two variances shows that the SD evaluation determined by including the covariances is almost twice as large as that evaluated by variances only. Considering that the demonstrated computation is similar to extracting elevation from DTM, it can be seen that ignoring the covariances can produce optimistic, but wrong, evaluations.

DTM APPLICATIONS

Unlike other GIS layers that represent the accurate position of each element, the DTM layer represents only the shape of the terrain (and not the altitude of every point). Therefore, when using the term accuracy for DTM, questions arise about some of the qualitative and quantitative aspects, at times mixed together. While the quantitative aspect is related to the accuracy estimate of each extracted point, the qualitative aspect is mainly related to determining a representative model of the terrain. Many methods have been suggested for evaluating the qualitative aspect. For example, Doytsher and Shmutter (1982) found profile collection to be an adequate way of representing the terrain, disapproving of grid collection due to its rigid character. Theobald (1989) suggests contour collection combined with collection of hydrologic features. Blace (1987) suggests terrain representations based on grid collection combined with break lines and the collection of variations from higher degrees like changes in curvature, etc. Li (1992) is checking the effect of adding break lines to a regular grid to show that the addition of break lines can reduce the grid's density.

The terrain representation model is the basic step for elevation extraction. This issue and the determination of its accuracy estimate is discussed next.

Accuracy Estimate of Computed Elevation

Schut (1976) presents and discusses elevation extraction strategies that vary from strategies for grid formation from irregular data. He suggests elevation extraction from a given data set. Two basic elevation extraction types can be identified: extraction from a regular grid and extraction from a TIN (triangular irregular network). While data is usually extracted from the TIN model by linear interpolation, elevation data extracted from a regular grid can be evaluated using up to 16 parameters (employing bicubic polynomials)—although usually 4-parameter (bilinear polynomial) interpolation is used.

Since all elevation extraction methods involve a transformation model, the accuracy estimate of the extracted data should refer to the model, so that by using the error propagation technique, the elevation's accuracy is estimated by: $\Sigma_h = \mathbf{F}_h * \Sigma_H * \mathbf{F}_h^T$.

For the bilinear interpolation, elevation is computed by:

$$h = a + b*(X - X_0) + c*(Y - Y_0) + d*(X - X_0)*(Y - Y_0) \qquad (15)$$

where a, b, c, d are determined by the elevations of the 4 corners—H_1, H_2, H_3, H_4. X_0, Y_0 = cell's origin coordinates. Then, the accuracy estimate for the elevations is found by

$$\sum_H = \begin{pmatrix} \sigma_{X_0}^2 & & & & & \\ & \sigma_{Y_0}^2 & & \text{COV} & & \\ & & \sigma_{H_1}^2 & & & \\ & & & \sigma_{H_2}^2 & & \\ & \text{COV} & & & \sigma_{H_3}^2 & \\ & & & & & \sigma_{H_4}^2 \end{pmatrix} \qquad (16)$$

The variance-covariance matrix, Σ_H, is derived from the variance-covariance matrix computed for the four corners forming the cell. The extracted elements refer to elevations of the corners and to X, Y coordinates of the origin, and contain their variances and the relevant covariances. The matrix **F** (at size of $\times 6$) is defined as

$$\mathbf{F}_h = \left(\frac{\partial h}{\partial X_0}\ \frac{\partial h}{\partial Y_0}\ \frac{\partial h}{\partial H_1}\ \frac{\partial h}{\partial H_2}\ \frac{\partial h}{\partial H_3}\ \frac{\partial h}{\partial H_4}\right) \quad (17)$$

This error propagation mechanism was formulated as the basis for evaluating DTM applications.

Accuracy Estimate Evaluation

Forming the accuracy estimate mechanism facilitates the evaluation of the affecting element. Table 7 presents a variance-covariance matrix of a given DTM cell. The matrix can be divided into three groups: the variance-covariance of the planimetric components, where the covariances are negligible; the covariances between the planimetric and the altimetric components, which are also negligible; and the variance-covariance between the altimetric components, which by all accounts are the dominant part of the matrix where the covariances display nearly 100% correlation and the variances are almost 1.5 times greater than the planimetric variances.

Such a dominant altimetric matrix gives rise to a question: what is the actual effect of the planimetric component? Two tests were conducted, one using a low slope of 5% along both axes and the second considering a 25% slope. Separating the horizontal effect from the vertical produced the following results: For the 5% slope the elevation accuracy estimate was $\Sigma_h = (0.003 + 1) * \sigma^2$, while for the 25% slope the result was $\Sigma_h = (0.08 + 1) * \sigma^2$.

Both results show the poor effect of the planimetric components on the accuracy estimate. Analysis of the components shows that the lack of correlation within the planimetric components, as well as between them and the altimetric elements that are highly correlated, is the reason for these results.

Another element can be added to the computed accuracy, which is related to the terrain characteristics. Ackermann (1978) defines three affecting elements: average shape, average wave length around the DTM cell, and the sampling density. Li (1993) suggests its evaluation using the following formula:

$$b * \left(1 + \frac{4 * S}{W}\right) * (S * \tan\alpha)^2$$

where b = 17/3,072—derived from an experimental test he performed; S = DTM cell size; α = average terrain slope; and W = average wave length.

Contour's SD Evaluation

Evaluation of the contours SD is essential for the determination of the vertical interval between adjacent lines. For example, an extraction of a 5 m

TABLE 7. Variance-Covariance Matrix of DTM Cell

7.95	0.24	1.83	1.85	1.86	1.85
0.24	8.28	0.86	0.90	0.90	0.90
1.83	0.86	12.34	12.39	12.35	12.33
1.85	0.90	12.39	12.45	12.41	12.39
1.86	0.90	12.35	12.41	12.38	12.35
1.85	0.90	12.33	12.39	12.35	12.33

interval contours from a DTM grid with 5 m precision will generally produce unreliable contours. The aim of the contour accuracy estimate is to define the tolerance (presented as a buffer) for each contour, and by overlaying them, to be able to determine a credible vertical interval.

Contour extracted from DTM is computed by interpolation techniques where the path is defined by a set of calculated points at the same elevation. The contour accuracy is, therefore, evaluated by the accuracy of the computed point (the SD of each point is determined by the mechanism described previously). The evaluated SD refers to the altimetric plane, while the desired tolerance refers to the planimetric plane. The translation of altimetric SD to its planimetric description is performed by relying on the relief shape (its slope). A function that mediates between the altimetric plane to the horizontal plane is defined by the interpolation function. The ratio Δ "elevation"/Δ "position" is expressed by

$$dx = \frac{dh}{b + d*(y - y_0)}, \quad dy = \frac{dh}{c + d*(x - x_0)} \tag{18}$$

Contour tolerances can be generated by converting the altimetric SD to a planimetric SD and by computing the gradient for each point. These tolerances are presented by buffer generation. Fig. 3 illustrates these buffers (represented by thin lines).

The generated buffers are not uniform in shape or characteristics. Comparisons with buffers that would have been generated by direct photogrammetric contour extraction, where the buffers would have parallel the contours, emphasizes this aspect. Figs. 3 and 4 illustrate the effect of the shape of the relief on the buffer characteristics.

Fig. 4(a) represents a window that was extracted from Fig. 3, where the terrain is characterized by a sharp slope. The buffer edges are parallel and relatively close to the contour path. These characteristics are explained by the sharp slope that transforms large differences in elevation to small differences in position, and the negligible effect of slope variation explains the edge's parallel shape. In contrast, Fig. 4(b) represents a window where the terrain is characterized by a moderate slope. The buffers here are wider than

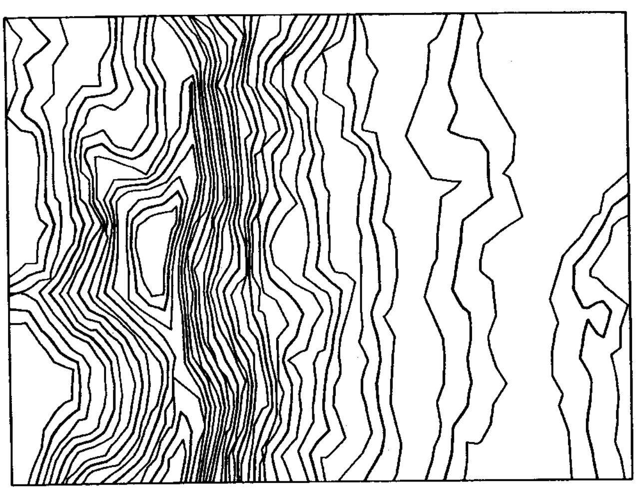

FIG. 3. Contours with SD Buffers

(a) (b)

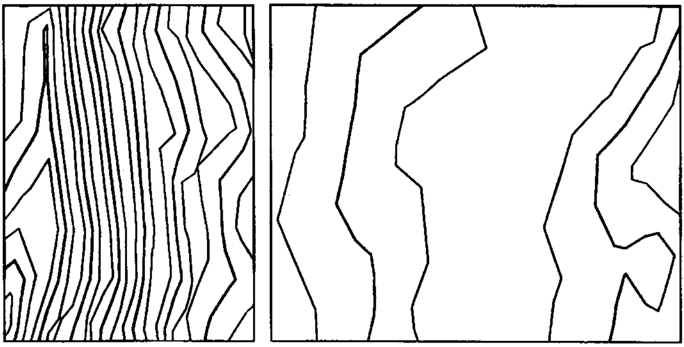

FIG. 4. Slope Effect on SD Buffers

those of Fig. 4(a) and their edges are more sensitive to slope variation, thus being not always parallel to the contour path.

SUMMARY

The paper presents a direct approach to accuracy estimate evaluation, and an analysis of the factors affecting ground coordinates accuracy. An analysis of the results has shown that ignoring the accuracy estimate of the measurements can lead to biased results. In addition it was shown that, due to the accuracy estimate variation throughout the model and the dominant values of covariances, no fixed value that represents the accuracy can be determined.

The implementation of the mechanism was presented for the accuracy estimates of both direct photogrammetric measurements and of applications based on the measurements. The implementation shows that, notwithstanding its mathematical complexity, the method can be easily implemented within GIS databases and could facilitate the precise evaluation of spatial data and spatial analysis. Due to the number of applications based on the DTM layer, special emphasis was devoted to demonstrating the mechanism implementation itself and on several derived applications.

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