

CAMERA CALIBRATION USING STRAIGHT LINES: ASSESSMENT OF A MODEL BASED ON PLANE EQUIVALENCE

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ABSTRACT

The aim of this paper is to present the derivation and experimental assessment of a camera calibration method based on control straight lines. A mathematical model using straight lines was derived to consider both interior and exterior orientation parameters as unknown. The Interior Orientation Parameters to be estimated are the camera focal length, the coordinates of the principal point and the lens distortion coefficients. The mathematical model is based on the equivalence between the vector normal to the projection plane in the image space and the vector normal to the rotated projection plane in the object space. This model was implemented and tests with real data were performed. The results obtained with the proposed model were compared with those obtained with the calibration with bundle method and it is shown that the results are similar for both models.

1. INTRODUCTION

Camera calibration is a fundamental task in Photogrammetry and Computer Vision, in which a set of Interior Orientation Parameters -IOPs (usually focal length, coordinates of the principal point, and lens distortion coefficients) are estimated (Brown, 1971), (Clarke and Fryer, 1998). This process can be carried out with laboratory methods, such as goniometer or multicollimator, or stellar and field methods, including mixed range field, convergent cameras and self-calibrating bundle adjustment. The lack of accurate IOPs leads to unreliable results in the photogrammetric process.

Orientation of digital images and camera calibration using lines have gained interest, basically due to the potential of automation and the robustness of some methods of line detection. Another remarkable advantage of using lines is the number of observations that can be provided, improving significantly the overall system redundancy.

In general, the field calibration and orientation methods solve the problem using points as control elements in a bundle adjustment. The non-linearity of the model and problems in point location in digital images are the main drawbacks of these approaches. The proposed solutions to overcome these problems include either the use of linear models (Lenz and Tsai, 1988) or the use of linear features: (Brown, 1971), (Lugnani, 1980), (Mulawa and Mikhail, 1988), (Haralick, 1989), (Liu et al., 1990), (Wang and Tsai, 1990), (Lee et al., 1990), (Tommaselli and Tozzi, 1996), (Quan and Kanade, 1997), (Habib et al., 2002), (Schenk, 2004). Some of these works deal only with the indirect determination of Exterior Orientation Parameters (EOPs). (Ethrog, 1984), however, presented a photogrammetric method for

determining the IOPs and the orientation angles, using objects with parallel and perpendicular lines, considering photographs taken with non-metric cameras.

One of the earliest approaches using lines in Photogrammetry was the plumb line calibration method (Brown, 1971). This method is suitable to recover the radial and decentring lens distortion coefficients, while the remaining interior (focal length and coordinates of the principal point) and exterior orientation parameters have to be determined by a complementary method (Fryer, 1996, p. 168). In the original derivation of the plumb line method, Brown considered the principal point coordinates as unknowns, but critical correlations with the decentring distortion parameters were observed (Fryer, 1996). A similar method was presented by Prescott and Mclean (1997) who compared their line-based method of estimation of lens distortion coefficients with the point based linear method of Tsai (Lenz and Tsai, 1988) and reported similar results for both methods showing the potential accuracy of line-based approaches. Tommaselli and Telles (2006) proposed a model based on the equivalence between projection planes, which will be presented and assessed with real data in this paper. A similar approach was also used to estimate orientation parameters of satellite pushbroom images (Tommaselli and Medeiros, 2010).

Another category of techniques for camera calibration uses vanishing points and lines. Grammatikopoulos et al. (2007) presented a technique for the automatic estimation of interior orientation parameters (camera constant, principal point coordinates, and coefficients of radial lens distortion) from images with three vanishing points of orthogonal directions. The authors considered the principal point and the point of best symmetry of distortion coincident. In the field of Computer Vision there are several approaches that use vanishing points and lines. A recent technique was proposed by He and Li (2008), using vanishing points in each image to estimate the interior orientation parameters. Additional steps involve the determination of the rotation matrix from the vanishing and image edges and the determination of the translation matrix.

Several other available methods using lines consider only the EOPs estimation, with no mention to the simultaneous determination of the IOPs, which are considered known from previous calibration.

The digital cameras produced for the consumer market differ in size, cost and stability of the interior geometry in comparison to the metric analogue cameras. As a consequence, when using digital non-metric cameras for metric purposes, it is recommended to use on-the-job calibration methods, or rely on periodic calibrations. Calibration fields with straight lines are easy to build, facilitating periodic calibrations of digital cameras.

The aim of this paper is to present a mathematical model relating the image and object spaces using straight lines, considering the IOPs as unknown. This model was previously presented and tested with simulated and real data (Tommaselli and Telles, 2006). In this paper, new experiments will be presented. The results obtained with the proposed model were compared with those obtained with the conventional calibration with the bundle model, based on the collinearity condition.

2. MATHEMATICAL MODEL

In this section the mathematical model using straight lines will be derived considering both interior and exterior orientation parameters as unknowns. The IOPs to be estimated can be the

camera focal length, the coordinates of the principal point and the lens distortion coefficients (radial and decentring).

The mathematical model is based on the equivalence between the vector normal to the projection plane in the image space and the vector normal to the projection plane in the object space. This model is an expansion of the equivalent plane model, proposed by Tommaselli and Tozzi (1996) (more details can be found in Tommaselli and Telles, (2006)).

The projection plane contains the straight line in the object space, the projected straight line in the image space, and the perspective centre of the camera (PC) (Figure 1).

In the original equivalent plane model the vector \mathbf{n} , normal to the projection plane in the image space, was expressed as a function of the focal length and the θ - ρ parameters of the straight line in the photogrammetric reference system. In order to include the lens distortion parameters and the coordinates of the principal point in the model, the straight line in the image space has to be expressed by the coordinates of its endpoints.

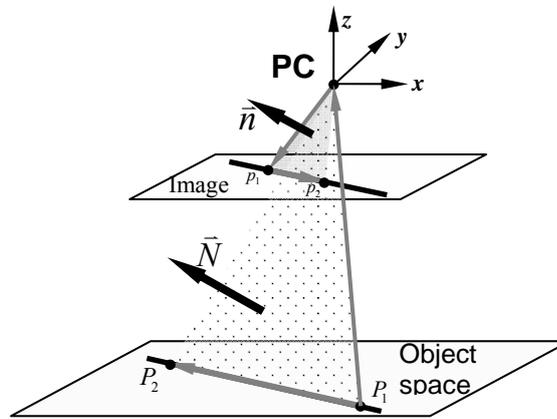


Figure 1: projection plane and normal vectors.

Considering the Conrady-Brown lens distortion model the coordinates of an image point in the photogrammetric reference system can be given by equations 1.

$$\begin{aligned} x &= x' - x_0 + \bar{x}(K_1 r^2 + K_2 r^4 + K_3 r^6) + P_1(r^2 + 2\bar{x}^2) + 2P_2\bar{x}\bar{y} \\ y &= y' - y_0 + \bar{y}(K_1 r^2 + K_2 r^4 + K_3 r^6) + P_2(r^2 + 2\bar{y}^2) + 2P_1\bar{x}\bar{y}, \end{aligned} \quad (1)$$

where:

- x, y are the refined photogrammetric coordinates;
- x', y' are the observed point coordinates in a centred image system;
- K_1, K_2, K_3 are the radial lens distortion coefficients;
- P_1, P_2 are the decentring distortion coefficients;
- x_0, y_0 are the coordinates of the principal point in the centred image system;
- $\bar{x} = x' - x_0$; $\bar{y} = y' - y_0$ and $r = \sqrt{\bar{x}^2 + \bar{y}^2}$.

The vector \mathbf{n} , normal to the projection plane in the image space, can then be written as the following vector product:

$$\vec{n} = \begin{bmatrix} \Delta x_{12} \\ \Delta y_{12} \\ 0 \end{bmatrix} \wedge \begin{bmatrix} x_1 \\ y_1 \\ -f \end{bmatrix} = \begin{bmatrix} -f\Delta y_{12} \\ f\Delta x_{12} \\ x_2 y_1 - x_1 y_2 \end{bmatrix}, \quad (2)$$

where:

$$\Delta x_{12} = x_2 - x_1;$$

$$\Delta y_{12} = y_2 - y_1;$$

f is the camera focal length.

The vector \vec{N} , normal to the projection plane in the object space, is defined by the vector product of the direction vector ($P_2 - P_1$) of the straight line and the vector difference (PC - P_1) (See Figure 1).

$$\vec{N} = \begin{bmatrix} \Delta X_{12} \\ \Delta Y_{12} \\ \Delta Z_{12} \end{bmatrix} \wedge \begin{bmatrix} \Delta X_{01} \\ \Delta Y_{01} \\ \Delta Z_{01} \end{bmatrix} = \begin{bmatrix} \Delta Y_{12}\Delta Z_{01} - \Delta Y_{01}\Delta Z_{12} \\ \Delta X_{01}\Delta Z_{12} - \Delta X_{12}\Delta Z_{01} \\ \Delta X_{12}\Delta Y_{01} - \Delta X_{01}\Delta Y_{12} \end{bmatrix}, \quad (3)$$

where:

$$\Delta X_{12} = X_2 - X_1, \quad \Delta Y_{12} = Y_2 - Y_1, \quad \Delta Z_{12} = Z_2 - Z_1,$$

$$\Delta X_{01} = X_1 - X_0, \quad \Delta Y_{01} = Y_1 - Y_0, \quad \Delta Z_{01} = Z_1 - Z_0.$$

X_0, Y_0, Z_0 are the coordinates of the Perspective Centre (PC) of the camera in the object space reference system; X_1, Y_1, Z_1 and X_2, Y_2, Z_2 are the 3D coordinates of the object straight line endpoints.

Multiplying vector \vec{N} by the rotation matrix \mathbf{R} eliminates the angular differences between the object and the image reference systems resulting in a vector normal to the projection plane in object space that has the same orientation as vector \vec{n} , normal to the projection plane in the image space, but different in magnitude.

$$\mathbf{R} \cdot \vec{N} = \lambda \cdot \vec{n}, \quad (4)$$

where λ is a scale factor and \mathbf{R} is the rotation matrix defined by the sequence $R_z(\kappa) \cdot R_y(\varphi) \cdot R_x(\omega)$ of rotations.

$$R_{\omega, \varphi, \kappa} \begin{bmatrix} \Delta Y_{12}\Delta Z_{01} - \Delta Y_{01}\Delta Z_{12} \\ \Delta X_{01}\Delta Z_{12} - \Delta X_{12}\Delta Z_{01} \\ \Delta X_{12}\Delta Y_{01} - \Delta X_{01}\Delta Y_{12} \end{bmatrix} = \lambda \begin{bmatrix} -f\Delta y_{12} \\ f\Delta x_{12} \\ x_2 y_1 - x_1 y_2 \end{bmatrix}. \quad (5)$$

Expanding (5) gives:

$$\begin{aligned} r_{11}N_1 + r_{12}N_2 + r_{13}N_3 &= -\lambda f\Delta y_{12} \\ r_{21}N_1 + r_{22}N_2 + r_{23}N_3 &= \lambda f\Delta x_{12} \\ r_{31}N_1 + r_{32}N_2 + r_{33}N_3 &= \lambda(x_2 y_1 - x_1 y_2) \end{aligned} \quad (6)$$

In order to eliminate the element λ , the first and the second equations of (6) are divided by the third one, giving:

$$\begin{aligned}
(x_2 y_1 - x_1 y_2)(r_{11} N_1 + r_{12} N_2 + r_{13} N_3) + f \Delta y_{12} (r_{31} N_1 + r_{32} N_2 + r_{33} N_3) &= 0 \\
(x_2 y_1 - x_1 y_2)(r_{21} N_1 + r_{22} N_2 + r_{23} N_3) - f \Delta x_{12} (r_{31} N_1 + r_{32} N_2 + r_{33} N_3) &= 0
\end{aligned} \tag{7}$$

By substituting the distortion equations (1) into (7) gives the final model.

In the original equivalent planes model (Tommaselli and Tozzi, 1996) there was a need to explicitly isolate the observations, in order to use the Iterated Extended Kalman Filtering, in a sequential approach. Owing to this need and to avoid divisions by zero, two groups of equations were derived, depending on the line orientation in the image. In the current case, a different strategy for deriving the model was used, because a more general estimation model was adopted (Mikhail, 1976), avoiding divisions and, therefore, generating just two equations suitable for lines of any orientation.

Although the model has two endpoints coordinates as observations, it is not required that these image endpoints should be correspondent to endpoints in the object space, thus preserving one of the main advantages of the line based models. A unified method of adjustment with weighted constraints to the parameters was used for parameter estimation giving a high degree of flexibility (Mikhail, 1976). The estimation method was implemented in C++ language. A conventional bundle adjustment based on the collinearity equations was also used as a reference method.

A linear feature extraction method aiming at subpixel precision was implemented based on the fitting of a parabolic surface to the grey level values of neighboring pixels around the segmented line. The intersection of this surface with a normal plane to the line direction generates a parabolic equation that allows estimating the subpixel coordinates of the point in the straight line, assuming that this point is the critical point of this function. For each straight line, several small patches are analyzed generating a sequence of points in that line with subpixel precision (Bazan et al., 2005). It is important to note that when using the proposed method based on straight lines there is no need for point to point correspondence.

The circular targets image coordinates corresponding to the points were extracted with subpixel accuracy using an interactive tool that computes the centre of mass after automatic threshold estimation.

3. EXPERIMENTS AND RESULTS

The proposed method was implemented in C++ language and tested with real data. In this section some experiments and results are presented and discussed. The developed method was assessed through the comparison with the results achieved by the conventional calibration with bundle method based on the collinearity model, implemented in existing software (CC-Camera Calibration) (Galo, 1993).

A close range test field, with 15 black steel cables and 53 circular targets over a white wall was used (Figures 2 and 3). The object space coordinates of the circular targets in a local reference system were previously estimated with topographic methods (theodolite intersection) with an accuracy of 2 mm. The coordinates of two endpoints of each straight line were then estimated from the measurements of linear offsets between targets and points in the lines, obtained with a metallic scale, with an uncertainty around 1 mm.

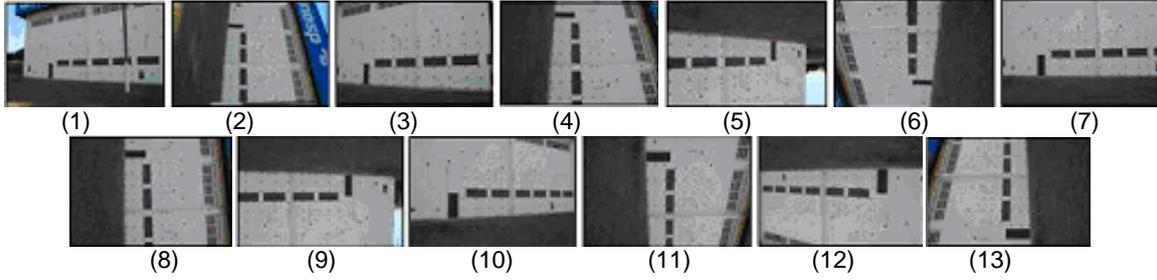


Figure 2: images used in the experiments with real data.

Thirteen images (six of them with 30° of convergence, and six with 90° rotation around z axis, as can be seen in Figure 2) were taken with a Sony DSC-R1 digital camera (10 megapixel) with a 26 mm focal length focused to the infinite (35 mm equivalent). Considering the 35 mm format, the pixel size for this camera is 0.009 mm.

To assess the accuracy of the proposed method with real data, a comparison with the field calibration based on bundle method was also performed. The 3D coordinates of control points were considered as weighted constraints, with a standard deviation of 3 mm. The comparison was based in the estimated parameters with both methods and in the statistics of discrepancies in the estimated coordinates of independent check points. As it was previously mentioned, the control and check points are circular targets, as shown in Figure 3.

Firstly, the camera was calibrated with the bundle method using 3D coordinates of 53 Ground Control Points (GCPs) as weighted constraints and 13 images, and these results were then used as a reference for comparison with the proposed method using lines. The computed IOPs and their estimated standard deviations are presented in Table 1. Several sets of interior orientation parameters were evaluated and the results were assessed to keep those with statistical significance. In this case, considering the accuracy of the GCP coordinates, only the parameter K_1 was significant for lens distortion modeling and it was used in the experiments.

Table 1: IOPs estimated with conventional bundle adjustment, considering 13 images and 53 GCPs.

Parameters	f (mm)	x_o (mm)	y_o (mm)	K_1 (mm^{-2})
Estimated Value	26.0997	0.0228	0.0411	-1.01×10^{-4}
Estimated Standard Deviation	0.0086	0.0021	0.0026	5.69×10^{-7}

Two endpoints for each straight line in each image were measured using a subpixel extraction technique (Bazan et al., 2005). It is not required that these points are correspondent to the endpoints of the same line in the other images.

In the second experiment, 13 images, 15 GCPs and 15 straight lines (see Figure 3) were used. The number of GCPs was reduced to achieve the same number of available straight lines.

In these experiments (with straight lines and with points) the following IOPs were considered: focal length, principal point coordinates and first order radial lens distortion coefficient (K_1). As previously mentioned, the decentring distortion coefficients (P_1 and P_2), and the higher order radial lens distortion coefficients (K_2 and K_3) were neglected after some preliminary experiments showing that their magnitudes were small and incompatible with the quality of the observed image coordinates.

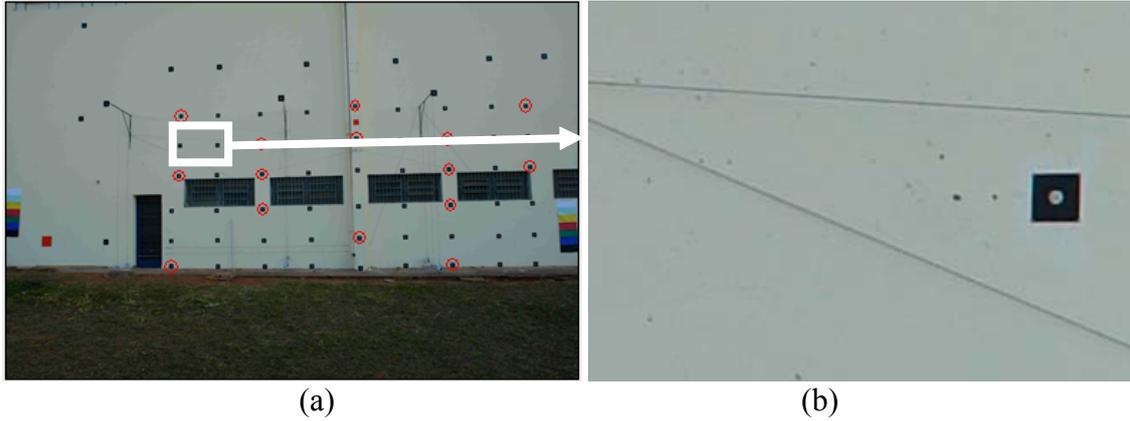


Figure 3: (a) terrestrial test field with GCPs used in the second experiment; (b) an extract showing one circular target and part of two horizontal straight lines.

Table 2 presents the IOPs and the EOPs of the first image, which were estimated using 15 straight lines (second and third columns) and with 15 GCP (fourth and fifth columns); in the last two columns the discrepancies related to the same parameters computed by the bundle method with 53 GCPs are presented.

Table 2: IOPs and the EOPs of the first image, estimated using straight lines and Ground Control Points.

Parameter	Calibration using 15 straight lines		Calibration using 15 Points (bundle method)		Discrepancies with reference to the bundle method (53 GCP)	
	Estimated Value	Estimated Standard Deviations	Estimated Value	Estimated Standard Deviation	Points	Straight Lines
f (mm)	26.1470	1.56×10^{-02}	26.1900	2.16×10^{-02}	-0.0473	-0.0903
x_0 (mm)	0.0213	3.37×10^{-03}	0.0247	6.23×10^{-03}	0.0015	-0.0019
y_0 (mm)	0.0408	4.28×10^{-03}	0.0692	5.40×10^{-03}	0.0003	-0.0281
K_1 (mm ⁻²)	-1.06×10^{-04}	1.02×10^{-06}	9.93×10^{-05}	1.63×10^{-06}	0.0002	0.0000
κ (rad)	-5.08×10^{-02}	1.49×10^{-04}	-5.19×10^{-02}	2.11×10^{-04}	-0.0002	0.0009
φ (rad)	3.97×10^{-01}	2.77×10^{-04}	3.97×10^{-01}	5.42×10^{-04}	-0.0005	-0.0011
ω (rad)	1.20×10^{-01}	4.39×10^{-04}	1.20×10^{-01}	6.91×10^{-04}	-0.0003	-0.0008
X_0 (m)	110.102	5.50×10^{-03}	110.128	9.46×10^{-03}	-0.0120	-0.0381
Y_0 (m)	401.229	5.37×10^{-03}	401.230	8.87×10^{-03}	0.0070	0.0056
Z_0 (m)	13.289	7.48×10^{-03}	13.314	1.06×10^{-02}	-0.0130	-0.0380

In general, the results have shown that the developed model is compatible to the conventional bundle method using control points. The basic hypothesis of a Qui-Square test was not rejected within a level of significance of 0.995 for both results (with control points and with straight lines). Tables 3 and 4 present the correlation coefficients among the IOPs and EOPs, computed from the covariance matrix of the estimated parameters for both methods (with straight lines - Table 3; and with points - Table 4).

It can be seen from the comparison and analysis of Table 3 and 4 that some correlations are slightly smaller in the estimation with the equivalent plane model with straight lines: focal length and Z_0 ; X_0 with Z_0 . On the other hand, correlation of Y_0 with ω was somewhat higher in the estimation with straight lines, probably due to the geometric configuration of the straight lines.

Table 3: correlations between IOPs and EOPs (first image) computed from the Covariance Matrix of the estimation method using 15 control straight lines and 13 images.

	X_0	Y_0	Z_0
f	0.82	-0.05	0.84
x_0	0.19	0.01	0.16
y_0	0.10	-0.05	0.23
K_1	-0.20	0.02	-0.23
κ	-0.14	0.21	-0.06
φ	0.76	0.26	-0.04
ω	-0.18	-0.95	0.04
X_0		0.14	0.50

Table 4: correlations between IOPs and EOPs (first image) computed from the Covariance Matrix of the estimation method using 15 control points and 13 images.

	X_0	Y_0	Z_0
f	0.81	-0.02	0.90
x_0	0.14	0.02	0.00
y_0	0.12	0.01	0.10
K_1	0.00	0.08	0.15
κ	-0.07	0.18	0.04
φ	0.75	-0.05	0.08
ω	-0.01	-0.90	-0.16
X_0		-0.03	0.61

These results are valid for the case studied and different values can arise in other configurations of straight lines and camera stations. Considering that the results are similar, an analysis with independent check points was also performed. The aim of the following orientation procedure was to determine which group of IOPs (either generated by the bundle method with GCPs or with straight lines) provides better results in the computation of EOPs and coordinates in the object space. To achieve that, a double bundle adjustment was performed, considering the IOPs as constant values (absolute constraints) and computing the EOPs and coordinates of 10 check points as unknown.

Two convergent images (1 and 10) and four well distributed control points were used to compute the 12 EOPs and the 10 check point 3D coordinates. The discrepancies in the coordinates of the 10 check points were estimated and, then, the RMSE (Root Mean Square Error), the average and the standard deviations were calculated. These results are presented in Table 5.

Table 5: mean error, standard deviations and RMSE for the discrepancies in the coordinates of independent check points when using IOP calculated with methods using points (bundle) and straight lines with 13 images.

Coord.	Mean error (mm)		Standard deviations (mm)		RMSE (mm)	
	Points	Straight Lines	Points	Straight Lines	Points	Straight Lines
X	0.33	0.19	1.38	1.50	1.43	1.51
Y	-0.16	-0.09	1.05	1.24	1.07	1.25
Z	0.60	0.67	2.31	2.27	2.40	2.37

Considering the RMSE presented in Table 5 (last two columns) it can be inferred that the orientation procedure with IOPs computed with both calibration methods produced similar results, within the accuracy (1-3 mm) of the check point coordinates determined by the field survey. The mean value of the errors in the XY coordinates of the check points are smaller when using the IOPs computed with the straight lines, whilst the standard deviations and RMSE of XY coordinates are slightly higher, although these variations are not significant.

A third experiment was performed with the same set of 15 Ground Control Points and 15 straight lines previously used, but using only 4 images (images 01, 07, 08 and 10 depicted in Figure 2). These images were chosen to assess a practical reduced set of images while avoiding correlations between parameters: two are normal (7 and 8), with image 8 being rotated 90° and two are near convergent (images 1 and 10, with a convergence around 23°).

The same procedures were applied, estimating two sets of IOPs and then estimating the EOPs with a double bundle adjustment using those IOPs previously estimated either with GCP or with straight lines. The statistics of the results in the check points coordinates are presented in Table 6.

Table 6: mean error, standard deviations and RMSE for the discrepancies in the coordinates of independent check points when using IOP calculated with methods using points (bundle) and straight lines with 4 images.

<i>Coord</i>	<i>Mean error (mm)</i>		<i>Standard deviations (mm)</i>		<i>RMSE (mm)</i>	
	<i>Points</i>	<i>Straight Lines</i>	<i>Points</i>	<i>Straight Lines</i>	<i>Points</i>	<i>Straight Lines</i>
<i>X</i>	0.26	0.40	1.51	1.32	1.53	1.38
<i>Y</i>	-0.37	-0.09	1.35	1.02	1.45	1.02
<i>Z</i>	0.60	0.46	2.16	2.10	2.24	2.15

It can seem from Table 6, that the results obtained with IOPs estimated with straight lines are slightly better than those obtained with IOPs estimated with GCP, with this reduced set of 4 images. All values of Mean Errors, Standard deviations and RMSE are smaller for straight lines, except the X coordinate Mean error.

Further experiments were performed to assess the behavior of both methods, and the results were similar, indicating that the proposed method with straight lines produces similar results when compared to the conventional calibration with the bundle adjustment, and in some specific situations, provided better results.

4. CONCLUSIONS

The results obtained with the proposed method that uses straight lines are statistically comparable with the conventional calibration with the bundle method using control points.

It is important to remember, however, that in the experiment the authors still did not take advantage of the potential redundancy provided by the straight lines. For each straight line only two equations were generated (two endpoints for each straight line) although there is a potential for using several pairs of equations. The potential for multiple line measurements and its impact in the final results is still a topic to be studied in future work.

It is still worth to note the advantages of line based calibration: flexibility; no need for point to point correspondence and; lines extraction with subpixel accuracy. Also, in the proposed approach, IOPs and EOPs are simultaneously estimated.

Further topics of interest for future work are: the combination of points and lines, taking advantage of the rigidity of points and the redundancy provided by lines; the impact of the geometric configuration of straight lines (length, orientation and position in the images) in the parameters estimation and estimation of the full set of distortion coefficients.

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