Camera Calibration

EN193s08 3D Photography
Brown Fall 2003
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Geometric Camera Calibration

Augmented pin-hole camera model

- Focal point, orientation
- Focal length, aspect ratio, center, lens distortion

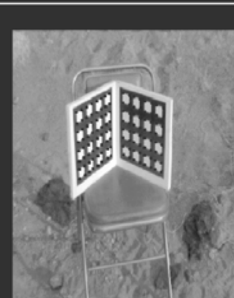
2D ⇔ 3D correspondence
"Classical" calibration

2D ⇔ 2D correspondence
SFM, "Self-calibration"

Camera Calibration

- Geometry
  - Where is the camera located ?
  - How is the camera oriented ?
  - What are the internal parameters ?
- Radiometry
  - What is the relation from incident light intensity to pixel values ?
- Parameters are estimated by observing objects of known dimensions and measuring their projections on the image plane

Calibration Patterns



Calibration grid
Z. Zhang, Microsoft Research

Chromaglyphs
Bruce Culbertson, HP-labs

Calibration From 2D Motion

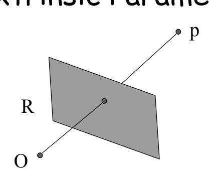
Structure From Motion (SFM)

- Track points over a sequence of images
- Solve for 3D positions and camera positions
- Calibrate internal parameters beforehand

Self-Calibration

- Solve for internal *and* external parameters
- E.g., [Polleyfeys, 98]

Extrinsic Parameters



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} p_x - O_x \\ p_y - O_y \\ p_z - O_z \end{bmatrix}$$

3 DOF Rotation (R=R(v)) + 3 DOF Position (*O*)

Intrinsic Parameters Without lens distortion

$$u = f_u\,(x/z) + c_u$$
$$v = f_v\,(y/z) + c_v$$

2 DOF Focal Lengths (f)
2 DOF Center of Projection (O)

Intrinsic Parameters Without lens distortion

$$s\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f_u & 0 & c_u \\ 0 & f_v & c_v \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

2 DOF Focal Lengths (f)
2 DOF Center of Projection (O)

Projection Matrix

$$s\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f_u & 0 & c_u \\ 0 & f_v & c_v \\ 0 & 0 & 1 \end{bmatrix}}_{P_I}\underbrace{\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}\begin{bmatrix} p_x - O_x \\ p_y - O_y \\ p_z - O_z \end{bmatrix}}_{P_E \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}}$$

Linear Geometric Calibration

$$s\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix}}_{P}\begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$$

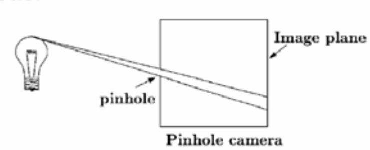
- Know 3D coords and 2D coords
- 11 unknowns (up to scale)
- 2 equations per point (eliminate s)
- 6 points are sufficient

Nonlinear Methods

- Problems with Linear Method
  - Too many free parameters
  - Doesn't model lens distortion
- Nonlinear Methods [Tsai, 1985]
  - Parameterize matrix P as a function of
    - Rotation vector v
    - Position vector O
    - Focal length vector f
    - Center of projection c
    - Lens distortion parameters
  - Use non-linear Least-Squares Solver

The pinhole camera

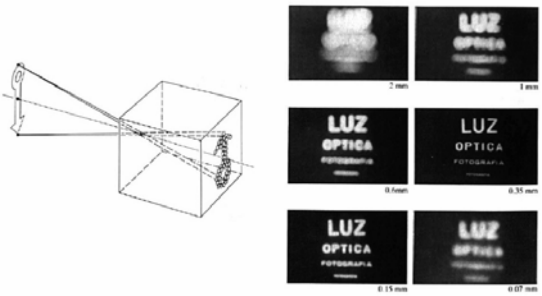
The first camera - "camera obscura" - known to Aristotle.



Small aperture = high fidelity
but **requires long exposure or bright illumination**

Pinhole camera

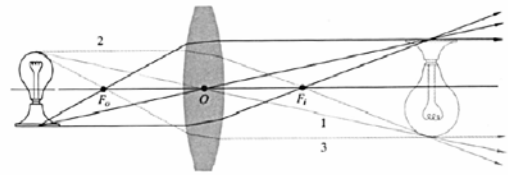
If aperture is too small, then diffraction causes blur.



[Figure from Hecht87]

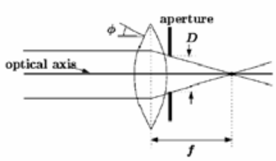
Lenses

Lenses focus a bundle of rays to one point.
=> can have larger aperture.



[Figure from Hecht87]

Lenses



A lens images a bundle of parallel rays to a focal point at a distance, *f*, beyond the plane of the lens.
Note: *f* is a function of the index of refraction of the lens.

An aperture of diameter, *D*, restricts the extent of the bundle of refracted rays.

Lenses

For economical manufacture, lens surfaces are usually spherical.

A spherical lens is behaves ideally if *Φ* is small:

$$\sin\phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \ldots \approx \phi$$

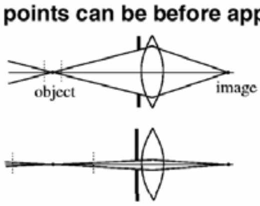
The angle restriction means we consider rays near the optical axis -- "paraxial rays."

Depth of field

Lens systems do have some limitations.

First, points that are not in the object plane will appear out of focus.

The *depth of field* is a measure of how far from the object plane points can be before appearing "too blurry."



Monochromatic aberrations

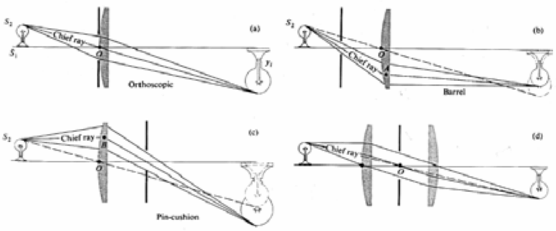
Allowing for the next higher terms in the sin*Φ* approximation:

$$\sin\phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \ldots \approx \phi - \frac{\phi^3}{3!}$$

...we arrive at the third order theory. Deviations from ideal optics are called the *primary* or *Seidel aberrations*:

- **Spherical aberration**
- **Coma**
- **Astigmatism**
- **Petzval curvature**
- **Distortion**

Distortion



[Figures from Hecht87]

Chromatic aberration

Cause:

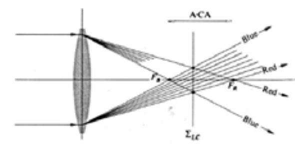
Index of refraction varies with wavelength.

Effect:

Focus shifts with color, colored fringes on highlights

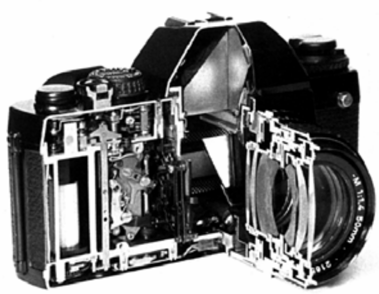
Ways of improving:

Achromatic designs



[Figure from Hecht87]

The art of optical design...

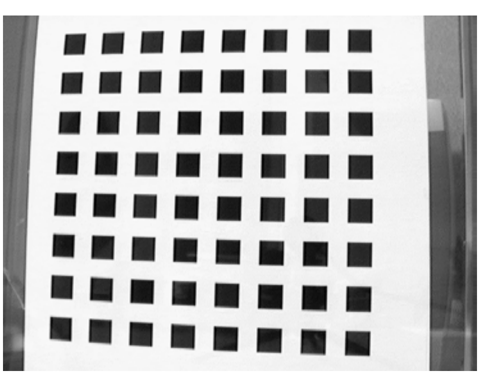


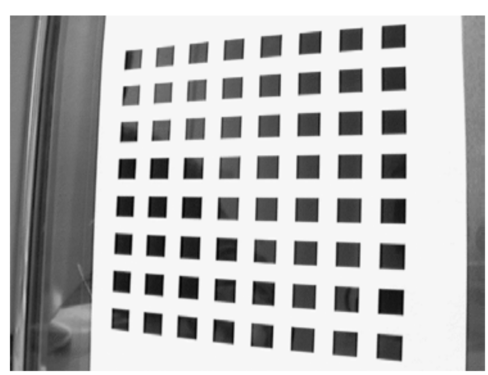
[Figure from Goldberg92]

Intrinsic Parameters With Lens Distortion

$$u = f_u \; \varphi_u(x/z, y/z) + c_u$$
$$v = f_v \; \varphi_v(x/z, y/z) + c_v$$

2 DOF Focal Lengths (f)
2 DOF Center of Projection (O)
? DOF Lens Distortion Function (φ)





Intrinsic Parameters With Lens Distortion

$$u' = f_u\ (x/z) + c_u$$
$$v' = f_v\ (y/z) + c_v$$

$$u = \rho_u\ (u', v')$$
$$v = \rho_v\ (u', v')$$

If lens distortion is moved to the last step,
It can be estimated independently as the first step,
And images can be warped after capture to remove distortion

Why is lens distortion independent ?

- Projection is now
  - a projective transformation from 3D to 2D
  - Followed by a 2D to 2D warping (lens distortion)
- Warping can be estimated and corrected by looking at the image of a planar pattern or lines because
- Homography from plane containing the pattern to image plane preserves straight lines
- So, the problem is to find the warping parameters that straighten all the lines as much as possible