Practical Paper

C-Factor for Softcopy Photogrammetry

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Abstract

The C-factor is an empirical value based on the precision of the photogrammetric instrumentation. The conventional C-factor has been used successfully over the years to determine the flying height required to produce a specified contour interval. C-factors for conventional instruments range from 900 to 2200 with 2200 being typical of state-of-the-art analytical plotters. Today's trend away from analog and computer-assisted plotters to digital photogrammetric workstations calls for a C-factor to use when the photogrammetry is to be accomplished using softcopy photogrammetric workstations. The soft C-factor will be based on conventional mathematics developed for analog instruments and related to today's cameras, scanners and soft copy workstations. Typical soft C-factors for digital photogrammetric workstations, assuming standard aerial photography and different scan spot sizes, are in a range from 800 to 2200.

Introduction

For decades, photogrammetrists in the United States have relied on an empirical value called the C-factor to determine the appropriate flying height for aerial photography when the desired map contour interval is specified. The C-factor is the dimensionless ratio of the flying height above ground to the contour interval (CI) that can be reliably plotted using the photography. In equation form,

$$\text{C-factor} = H/\text{CI} \tag{1}$$

where H is the flying height above ground and CI is the contour interval.

Manufacturers commonly state nominal C-factors for their photogrammetric instruments. They vary from 1200 to 2200 depending on the precision of the instruments. Table 1 shows nominal C-factors for some commonly used analog and computer-assisted instruments.

Contour accuracy depends not only on the plotting instrument, but also upon the nature of the terrain, the camera and its calibration, the resolution quality of the photography, the density and accuracy of the ground control, and the capability of the plotter operator. This has remained true for analog plotters, and is still applicable during the transition years to analytical, computer-assisted plotters. These conditions all combined to yield a total system C-factor, which assumes typical values for all these variables.

Now that the mapping profession has transitioned from computer-assisted plotters to digital photogrammetric workstations, it seems important to have a C-factor for the softcopy era. A soft C-factor based on rationale the same as before will be useful for relating flying height H to the contour interval CI that can be achieved with a softcopy workstation.

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Table 1. C-Factors for Photogrammetric Instruments

Instrument	C-Factor
Kelsh	1200
B-8	1300
PG-2	1600
AS-11	2000
Intermap	2200
LH Systems	2200
Zeiss P-1	2200

Mathematical Basis for Soft C-Factor

Because the C-factor is basically an expression of precision for the entire system, then it seems important to examine the fundamental parameters that affect precision in stereo photogrammetry.

The ability of a stereophotogrammetric mapping system to discriminate increments or errors in elevation is expressed by the well-known parallax equation derived by Hallert (1960) and utilized by Doyle (1963) with different notations to estimate precision in photogrammetry for near-vertical photography.

Assuming that the precision of point measurement is identical on both photos of the stereopair, i.e., $dx_1 = dx_2 = m_x$, the precision of a single observation for elevation may be expressed as

$$m_h = \frac{H}{f} \times \frac{H}{B} \times \sqrt{2}\, m_x \tag{2}$$

where f is the camera focal length, H is the flying height above ground, B is the base distance between two exposures, m_x is the precision of measurement on a single photo image, and H/B is the reciprocal of the base-to-height ratio.

As Doyle (1963) has pointed out, for any system, the limiting value of image measurement is a function of the linear resolution of the photography. Gardner (1932) has stated that the probable error of a single setting is one-fifth to one-sixth of the distance between two lines (one line pair) which are just resolved. Expressing Gardner's statement statistically and using line pairs (lp) per millimeter, yields an equation for precision of measurement related to resolution: i.e.,

$$0.675\, m_x = 1/5 \text{ mm/lp}$$

or

$$m_x = 0.3 \text{ mm/lp}. \tag{3}$$

Changing Equation 3 to use lines/mm (l/mm) instead of line

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pairs/mm (lp/mm) for digital terminology, Gardner's Equation 3 can be re-stated as

$$0.675\ m_x = 1/2.5\ \text{mm/l}$$

so that

$$m_x = 0.6\ \text{mm/l}. \tag{3a}$$

If all other errors in the system are assumed to be lumped into m_x, the contouring ability is directly related to this standard deviation m_x because, in the operation of contouring in a stereo model, each point is observed only once as it is passed.

In order to meet the criterion that 90 percent of elevations be correct within one-half the contour interval, National Map Accuracy Standards can be written as

$$1.64\ m_h = 0.5\ \text{CI}$$

so then

$$\text{CI} = 3.3\ m_h. \tag{4}$$

Substituting Equation 2 into Equation 4 yields

$$\text{CI} = 4.7\ \frac{H}{f} \times \frac{H}{B}\ m_x. \tag{5}$$

Equation 5 may be related to the usual concept of C-factor defined as the ratio of H/CI. Then an equation for C-factor, as used over the years with analog instruments, can be found from Equation 5: i.e.,

$$\text{C-factor} = \frac{H}{\text{CI}} = 0.21 \times \frac{B}{H} \times \frac{f}{m_x}. \tag{6}$$

Recognizing that the B/H ratio and the focal length f are part of the system geometry, it is apparent that variations of C-factor among photogrammetric instruments depend largely upon the capability of the instruments to utilize the resolution of the photography for precision measurement. That is, assume that all other errors are included in the measuring error, m_x, and that m_x is directly related to the resolution of the system. Therefore, Equation 6 can be exploited to derive a soft C-factor equation that is applicable to digital photogrammetric workstations.

First, to demonstrate the practical application of Equation 6, consider an analytical plotter utilizing conventional 152.4-mm focal-length mapping photography with B/H = 0.6. Thirty lp/mm is probably a reasonable estimate of the average resolution which can be utilized by the optical system of the plotter. Then, from Equation 3,

$$m_x = 0.3\ \text{mm}/30\ \text{lp} = 0.010\ \text{mm}$$

and, from Equation 6,

$$\text{C-factor} = 0.21 \times 0.6 \times 152.4\ \text{mm}/0.010\ \text{mm}$$

or

$$\text{C-factor} \cong 1920.$$

It is interesting to observe that the least count on most first-order plotters is 0.010 mm, and this yields a C-factor of 1920 that is sufficiently close to 2000 as generally claimed by the plotter manufacturers. Utilizing the work of Hallert (1960), Doyle (1963), and Gardner (1932), it has been shown that Equations 3, 3a, and 6 can be used to relate resolution with measuring precision, and measuring precision with a C-factor. Now it seems reasonable to look at the resolution in the softcopy image chain (pixels) and produce an analogous term called "Soft C-Factor" for the coming era in softcopy photogrammetry.

Soft C-Factor

The objective is merely to evaluate the components of the softcopy imaging chain and compute a system resolution that the digital workstation is utilizing. Then, enter the softcopy precision m_x into Equation 6 and a soft C-factor can be computed.

Total System Resolution (R_t)

Again, recognizing that the C-factor is an empirical value based on precision of measurement, the following expression can be utilized to evaluate each component. Then, using each component of the image chain, compute a total system resolution R_t that can be converted to m_x by Equation 3a. Finally, use m_x in Equation 6 to arrive at the soft C-factor.

The expression for total system resolution R_t (Meier, 1984; Light, 1996) is

$$1/R_t^2 = 1/R_f^2 + 1/R_s^2 + \ldots \tag{7}$$

where all values for Equation 7 must be in l/mm for softcopy, R_f is the resolution of the film in lp/mm converted to l/mm: i.e., 1 lp = 2 l, and R_s is scan spot size converted to l/mm.

Example: Assume a modern aerial film camera yields 40 lp/mm (80 l/mm) resolution to the user. Because 40 lp/mm is equivalent to 25 µm/lp, the appropriate scan spot size (SSS) should be 25 µm/lp × 1 lp/2.2 l = 11 µm/l = 11 µm/pixel. For converting lp to pixels, the rationale given by Larson and Wertz (1993) shows that 1 lp ≅ 2.2 pixels is appropriate to use when converting analog data to digital data. Eleven µm pixels will approximately preserve the 40 lp/mm (80 l/mm) resolution of the original film. Using the appropriate values for R_f and R_s in Equation 7, compute the total system resolution R_t: i.e.,

$$1/R_t^2 = 1/80^2 + 1/90^2$$

because R_f = 40 lp/mm (80 l/mm) and R_s = 1000 µm/mm/11 µm = 90 l/mm. Then, the total system resolution is

$$R_t = 59\ \text{l/mm}.$$

Table 2. Typical Soft C-Factors

Original Photos lp/mm (l/mm)	Scan Spot Size µm	l/mm	Eq (7) R_t µm	Eq (3a) m_x mm	Eq (6) Soft C-Factor
20 (40)	11	91	37	0.016	1200
20 (40)	15	67	34	0.017	1129
20 (40)	25	40	28	0.021	914
20 (40)	30	33	26	0.023	834
25 (50)	11	91	44	0.014	1371
25 (50)	15	67	40	0.015	1280
25 (50)	25	40	31	0.019	1010
25 (50)	30	33	28	0.022	872
30 (60)	11	91	50	0.012	1600
30 (60)	15	67	45	0.013	1477
30 (60)	25	40	33	0.018	1066
30 (60)	30	33	29	0.021	914
35 (70)	9	111	59	0.010	1920
35 (70)	11	91	55	0.011	1745
35 (70)	15	67	48	0.012	1600
35 (70)	25	40	35	0.017	1129
35 (70)	30	33	30	0.020	960
40 (80)	9	111	65	0.009	2133
40 (80)	11	91	60	0.010	1920
40 (80)	15	67	51	0.012	1600
40 (80)	25	40	36	0.017	1129
40 (80)	30	33	31	0.020	960

Research to validate these values by experiment should be conducted as soft-copy takes its place in the digital photogrammetry business.

Using the digital Equation 3a,

$$m_x = 0.6 \text{ mm}/59 \text{ l or}$$
$$m_x = 0.010 \text{ mm}.$$

Now, entering $m_x = 0.010$ mm into the C-factor Equation 6, one obtains Soft C-factor = 0.21 × 0.6 × 152.4 mm/0.010 mm, or Soft C-factor = 1920.

As an additional thought, it is recognized that the photogrammetric workstation's monitor plays a key role in presenting the stereo-model. Then, it follows that the monitor's dot pitch should be considered in computing an empirical C-factor. On the other hand, experiments by Wong (1997) show that monitor resolutions are fixed, regardless of scanned resolution or zoom ratios. In view of this and the need to keep computations simple and practical, the monitor's contribution is considered to be small and, therefore, is ignored in this derivation and is left for further research.

In summary, conventional 15/23 mapping camera photography with 40 lp/mm resolution, which was scanned at an 11-µm spot size, yields a softcopy C-factor that is slightly less than the C-factor for a first-order plotter. Although the soft C-factor is empirical, it can be shown that softcopy photogrammetry is capable of accomplishing precision topographic mapping and the soft C-factor can indicate the proper flying height. Table 2 gives computed soft C-factors for different photography resolution and useful scan spot sizes.

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