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BUNDLE ADJUSTMENT WITH SELF-CALIBRATION USING STRAIGHT LINES

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Abstract

Increased use of digital imagery has facilitated the opportunity to use features, in addition to points, in photogrammetric applications. Straight lines are often present in object space, and prior research has focused on incorporating straight-line constraints into bundle adjustment for frame imagery. In the research reported in this paper, object-space straight lines are used in a bundle adjustment with self-calibration. The perspective projection of straight lines in the object space produces straight lines in the image space in the absence of distortions. Any deviations from straightness in the image space are attributed to various distortion sources, such as radial and decentric lens distortions. Before incorporating straight lines into a bundle adjustment with self-calibration, the representation and perspective transformation of straight lines between image space and object space should be addressed. In this investigation, images of straight lines are represented as a sequence of points along the image line. Also, two points along the object-space straight line are used to represent that line. The perspective relationship between image- and object-space lines is incorporated in a mathematical constraint. The underlying principle in this constraint is that the vector from the perspective centre to an image point on a straight-line feature lies on the plane defined by the perspective centre and the two object points defining the straight line. This constraint has been embedded in a software application for bundle adjustment with self-calibration that can incorporate point as well as straight-line features. Experiments with simulated and real data have proved the feasibility and the efficiency of the algorithm proposed.

KEYWORDS: bundle adjustment, self-calibration, straight lines

INTRODUCTION

MOST PHOTOGRAMMETRIC APPLICATIONS are based on the use of distinct points. These points are often obtained from measurements in an analogue or digital environment. Recently, more attention has been paid to linear features. There are several motivations for the use of linear features in photogrammetry.

(a) Linear features are more useful than points for higher-level tasks such as object recognition.
(b) Automation of the map-making process is one of the major tasks in digital photogrammetry and cartography. It is easier to extract linear features from imagery automatically than distinct points (Kubik, 1991).
(c) Images of man-made environments are rich in linear features.

Straight-line constraints are incorporated into the bundle adjustment procedure by making use of the fact that the perspective transformation of a straight line is also a straight line. Mikhail and Weerawong (1994) proposed a straight-line constraint which ensures that a unit vector defining the object-space line, the vector from the perspective centre to a point on the object line and the vector from the perspective centre to the image point are coplanar. In their approach, the object line is represented as an infinite line segment. Additional literature concerned with straight lines in photogrammetric applications includes the work of Mulawa and Mikhail (1988), Tommaselli and Lugnani (1988), Ayache and Faugeras (1989), Tommaselli and Tozzi (1992), Habib (1998), van den Heuvel (1999a) and Tommaselli and Poz (1999). The majority of this literature defines straight lines in the object space as infinite lines using optimal (minimal) representation. Habib (1999) discussed various options for straight-line representation in the object space for photogrammetric applications. The primary representation considerations are uniqueness and singularities. Based on that discussion, it was determined that optimal representation of object space lines as infinite presents the following problems.

(a) For mapping purposes, line segments represent the major area of interest, rather than infinite lines.
(b) The associated error measures pertain to infinite lines. These measures might be completely different from those associated with line segments.
(c) Models involving infinite lines would require complicated perspective transformation algorithms between image and object space, making them difficult to incorporate in existing bundle adjustment programs.
(d) Finally, minimal representations always have singularities. They cannot represent all 3D lines in space.

In the research reported in this paper, two points along a line represent a straight line in object space. In this way, the line segment is well localised in object space, avoiding all the problems mentioned above. This representation of a straight line is attractive because such points can be easily introduced or obtained from a GIS database.

Previous photogrammetric research on linear features has focused primarily on aerial triangulation, without paying attention to the feasibility of using such features to perform self-calibration. Traditionally, calibration parameters are determined by means of bundle adjustment using a testfield with a large number of control points. Establishing and maintaining testfields is an expensive and specialised operation. With increasing demand for performing photogrammetric operations on imagery obtained from off-the-shelf digital cameras, the need to establish economic and efficient calibration techniques is growing. It is believed that using straight lines in the calibration process can result in accurate, reliable and more efficient self-calibration, which can be performed by users of digital cameras who are not photogrammetrists.

It should be noted that Brown (1971) developed the plumb-line method for camera calibration purposes using straight lines. Brown used deviations from straightness in images of object-space straight lines to derive radial and decentric lens distortions. This work is based on fitting straight lines through image lines using an adjustment procedure that solves for the principal point coordinates and radial and decentric lens distortion coefficients in addition to the line parameters. A further calibration procedure should be carried out to determine the principal distance and the systematic errors that took place during the image acquisition process. Distortions which are not corrected prior to applying the plumb-line method would contaminate the recovered lens distortion parameters.

Also, van den Heuvel (1999b) introduced a model for the recovery of the interior orientation parameters (IOPs) by applying constraints on straight-line measurements in a single image. This method is only applicable to images with parallel and perpendicular lines. Moreover, the IOPs are determined sequentially. Radial lens distortion is estimated first, followed by the determination of the principal point coordinates and the focal length.

The current research presents a new approach to using straight lines as well as distinct points in bundle adjustment with self-calibration. In the image space, straight lines are represented as a sequence of points along the image line. This representation is useful for the calibration process since it allows the distortions at each point along the image line to be incorporated. Other representations (for example, using polar parameters ρ, θ) assume that compensation has already been made for distortions. The authors believe that straight lines would have an improved impact on camera calibration, because the deformations and the distortions are mapped along their direction as a continuous function. In addition, users of digital cameras who are not photogrammetrists could easily implement the calibration process. The next section describes the most commonly used model for camera self-calibration. The mathematical model proposed, which relates image- and object-space straight lines, is then presented, followed by details of results obtained from implementation of this approach, using both synthetic and real data. Finally, conclusions and recommendations for future work are presented.

BACKGROUND: SELF-CALIBRATION MODEL

The main objective of photogrammetry is to invert the process of photography. When the film inside a camera is exposed to light, rays from the object space pass through the camera perspective centre (the lens) until they hit the focal plane (film) producing images of the photographed objects. The main mathematical model that is implemented in the majority of photogrammetric applications is the collinearity condition. The collinearity equations mathematically describe the fact that the object point, the corresponding image point and the perspective centre lie on a straight line.

During camera calibration, the IOPs are determined. They comprise the principal point coordinates, the principal distance (c) and the image coordinate corrections that compensate for various deviations from the collinearity model. There are four principal sources of departure from collinearity, which are "physical" in nature (Fraser, 1997). These are radial lens distortion, decentric lens distortion, image plane unflatness and in-plane image distortion. The net image displacement at any point is the cumulative influence of these perturbations. The relative magnitude of each one depends substantially on the nature of the camera being employed.

Radial lens distortion (Δx_{RLD}, Δy_{RLD}) is usually represented by polynomial series (1). The term K_1 alone will usually suffice in medium accuracy applications. The inclusion of the K_2 and K_3 terms might be required for higher accuracy and wide-angle lenses. The decision as to whether to incorporate one, two or three radial distortion terms can be based on statistical tests of significance such that

$$
\begin{aligned}
\Delta x_{RLD} &= K_1(r^2-1)x + K_2(r^4-1)x + K_3(r^6-1)x \\
\Delta y_{RLD} &= K_1(r^2-1)y + K_2(r^4-1)y + K_3(r^6-1)y
\end{aligned}
\tag{1}
$$

where $r=\sqrt{(x-x_p)^2+(y-y_p)^2}$, x_p and y_p are the image coordinates of the perspective centre and K_1, K_2 and K_3 are the radial lens distortion parameters.

A lack of centricity of lens elements along the optical axis gives rise to the second category of lens distortion, decentric distortion (Δx_{DLD}, Δy_{DLD}). The misalignment of the lens components causes both radial and tangential distortions which can be modelled by correction equations according to Brown (1966) as follows:

$$
\begin{aligned}
\Delta x_{DLD} &= P_1(r^2+2x^2) + 2P_2xy \\
\Delta y_{DLD} &= P_2(r^2+2y^2) + 2P_1xy
\end{aligned}
\tag{2}
$$

where P_1 and P_2 are the decentric lens distortion parameters.

Systematic image coordinate errors due to focal-plane unflatness can limit the accuracy of photogrammetric triangulation. Radial image displacement induced by focal-plane unflatness depends on the incidence angle of the imaging ray. Narrow-angle lenses of long focal length are much less influenced by out-of-plane image deformation than short focal length and wide-angle lenses. In order to compensate for focal-plane unflatness, the focal plane needs to be topographically measured. Then, a third- or fourth-order polynomial can model the resulting image coordinate perturbations.

In-plane distortions are usually manifested in differential scaling between x and y image coordinates. In addition, in-plane distortions introduce image axis non-orthogonality. Those distortions (Δx_{AD}, Δy_{AD}) are usually denoted as affine deformations and can be mathematically described by (3). It should be noted that affine deformation parameters that are correlated with other IOPs are eliminated (for example, shifts are eliminated since they are correlated with the principal point coordinates).

$$
\begin{aligned}
\Delta x_{AD} &= -A_1x + A_2y \\
\Delta y_{AD} &= A_1y
\end{aligned}
\tag{3}
$$

where A_1 and A_2 are the affine distortion parameters.

Traditionally, the self-calibration parameters are determined through bundle adjustment using a testfield with a large number of control points. Primitives with a higher level of abstraction (for example, straight lines and surface patches) have been used for various photogrammetric operations such as aerial triangulation and absolute orientation (Habib, 1999; Jaw, 1999). It is believed that using such primitives in the calibration process can result in accurate, reliable and more efficient self-calibration, which can be performed by users of digital cameras who are not photogrammetrists. The accuracy and reliability stem from the fact that distortion is a continuous phenomenon, which differs from one location to another throughout the image space. Therefore, a continuous curve (for example representing a straight line) can be used to continuously monitor the distortion along the curve rather than the recovery of the distortion parameters by observing discrete points.

In summary, this approach provides a more efficient and economic procedure for estimating the parameters involved in self-calibration models by using straight-line constraints rather than working with points. However, investigating such models is not the focus of this research.

MATHEMATICAL MODEL FOR INCLUDING STRAIGHT LINE CONSTRAINTS

Before explaining the mathematical model, it should be noted that the aim is to incorporate overlapping images with straight linear features and some tie and control points in a self-calibration process to estimate the following quantities:

(a) the exterior orientation parameters (EOPs) of the imagery involved;
(b) the IOPs of the cameras used;
(c) the ground coordinates of tie points; and
(d) the parameters defining the straight lines in the object space.

As depicted in Fig. 1, for a frame camera, a straight line in the object space will be a straight line in the image space in the absence of distortions. Deviation from straightness in the image space is a function of the distortion parameters. As mentioned before, inclusion of straight lines in the bundle adjustment procedure would require the answer to two main questions. First, what is the most convenient model for representing straight lines in the object space and image space? Second, how can the perspective relationship between image- and object-space lines be established? In this research, object-space lines are represented by two points along the line. These points are monoscopically measured in the one or two images within which this line appears. The relationship between these points and the corresponding object-space points is modelled by the collinearity equations. In the image

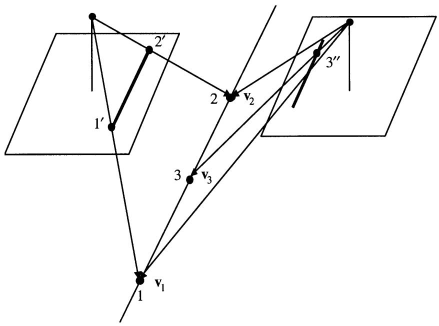

FIG. 1. 3D straight line in frame-camera imagery.

space, the lines will be defined by a sequence of intermediate points along the line. Once again, those points are monoscopically measured (there is no need to identify conjugate points in overlapping images). This representation is useful since it allows the distortions at each of these points to be individually modelled. The perspective relationship between image- and object-space lines is incorporated in a mathematical constraint. The underlying principle in this constraint is that the vector from the perspective centre to any intermediate image point along the line lies on the plane defined by the perspective centre of that image and the two points defining the straight line in the object space. In other words, the three vectors which are involved (Figs. 1 and 2):

(a) $\mathbf{v}_1$ (the vector connecting the perspective centre to the first point along the object space line),
(b) $\mathbf{v}_2$ (the vector connecting the perspective centre to the second point along the object space line), and
(c) $\mathbf{v}_3$ (the vector connecting the perspective centre to any intermediate point along the image line)

are coplanar so that

$$(\mathbf{v}_1 \times \mathbf{v}_2) \cdot \mathbf{v}_3 = 0. \quad (4)$$

Equation (4) incorporates the image coordinates of the intermediate point, EOPs, IOPs (which include the distortion parameters) as well as the ground coordinates of the points defining the object space line. The constraint in (4) can be written for any intermediate point along the line in the imagery. It should be noted that this constraint would not introduce any new parameters. The number of constraints is equal to the number of the measured intermediate points along the image line.

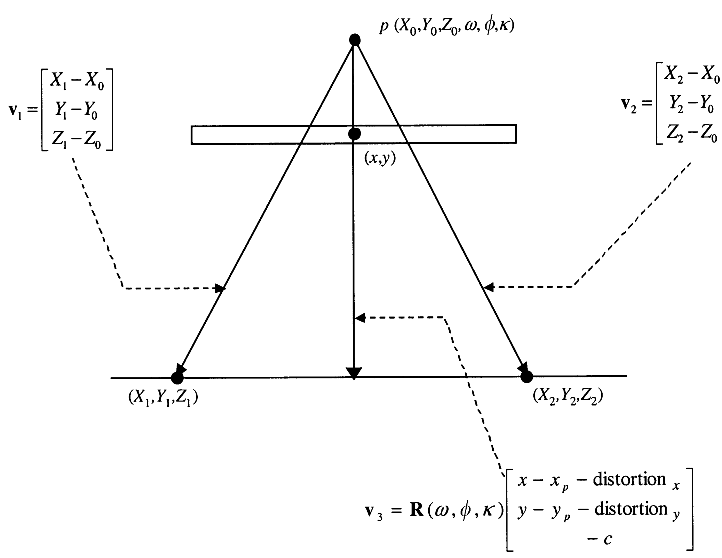

FIG. 2. Mathematical model of a straight line in frame-camera imagery.

In summary, for bundle adjustment with self-calibration using straight lines, the end points (points 1′ and 2′ in Fig. 1 for example) can be selected in any of the images where the straight line appears. These points need not be identifiable or even visible in other images. Four collinearity equations will be written using the measured end points for each line. The intermediate points (point 3″ for example) can be measured in any one of the overlapping images. These intermediate points need not be conjugate. A constraint is written for each intermediate point according to (4). Fig. 3 is a schematic drawing which clarifies the different scenarios for the end point selection. Fig. 3(a) shows a case where the end points of the straight line are selected in one image (Image 1), whereas in Fig. 3(b), they are selected in different images (Image 1 and Image 4). Intermediate points are shown in the same figure.

This approach can be extended to include higher-order primitives (for example, conic sections). It can also be applied to imagery from line scanners, the only difference being that the platform motion perturbations during the scene capture, as well as the distortion sources mentioned above, cause deviation from straightness in the imagery.

EXPERIMENTAL RESULTS

Synthetic Data

Six experiments have been carried out using synthetic imagery. In each case, five overlapping images have been generated with known EOPs and IOPs, including

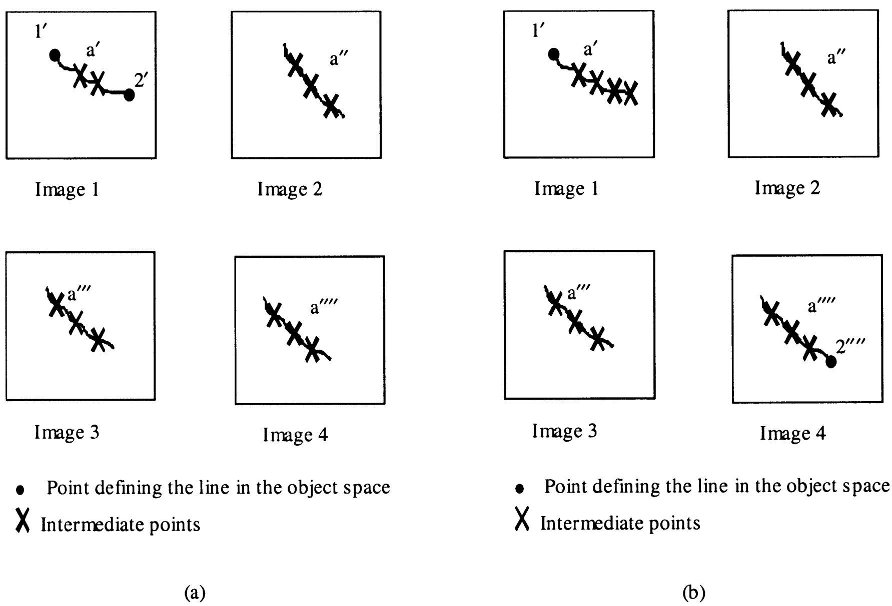

FIG. 3. Schematic drawing representing two examples for selection of the end and intermediate points in overlapping images.

the distortion parameters. The configuration specifications for these experiments are listed in Table I. Corresponding diagrams are given in Fig. 4, showing the footprint of the images, object space straight lines, measured distances, and tie and control points for the various experiments. Normally distributed noise, with a zero mean and 5 μm standard deviation, has been added to the image coordinates. Table II gives the results for the estimated IOPs, including the distortion parameters, for the different experiments compared to the true values. The absolute deviations of the estimated parameters from their true values are plotted in Fig. 5. The results from the first experiment show the best estimates for the IOPs when they are derived from traditional bundle adjustment with self-calibration using distinct control points. It can be seen that using straight lines for self-calibration, Experiment 2, yields as accurate results as for distinct points, if not better. It has to be noted that the number of control points in Experiment 2 is minimal. This means that straight-line constraints are the major factor for recovering the IOPs. The datum for Experiments 3 to 6 has been established by fixing the EOPs of one of the images to an arbitrary value, as well as fixing some distances in the object space. Throughout all these experiments, the IOPs have been accurately recovered using the straight-line constraints. Experiment 6 shows an extreme case where there are only two tie points (the distance between these points was implemented in the

TABLE I. Configuration specifications for Experiments 1 to 6 (synthetic data).

	Experiment number					
Parameter	*1*	*2*	*3*	*4*	*5*	*6*
Number of ground control points (±10 cm)	49	3	0	1	0	0
Tie points	None	6	9	8	9	2
Linear feature constraints	None	674 (8 lines)	674 (8 lines)	674 (8 lines)	674 (8 lines)	674 (8 lines)
Distance constraints (±10 cm)	None	None	1	1	6	1
Exterior orientation parameters	Unknown	Unknown	Fixed for one image (±10 cm, ±10″) Other images (unknown)	Fixed for one image (±10 cm, ±10″) Other images (unknown)	Fixed for one image (±10 cm, ±10″) Other images (unknown)	Fixed for one image (±10 cm, ±10″) Other images (unknown)
Interior orientation parameters	Unknown	Unknown	Unknown	Unknown	Unknown	Unknown

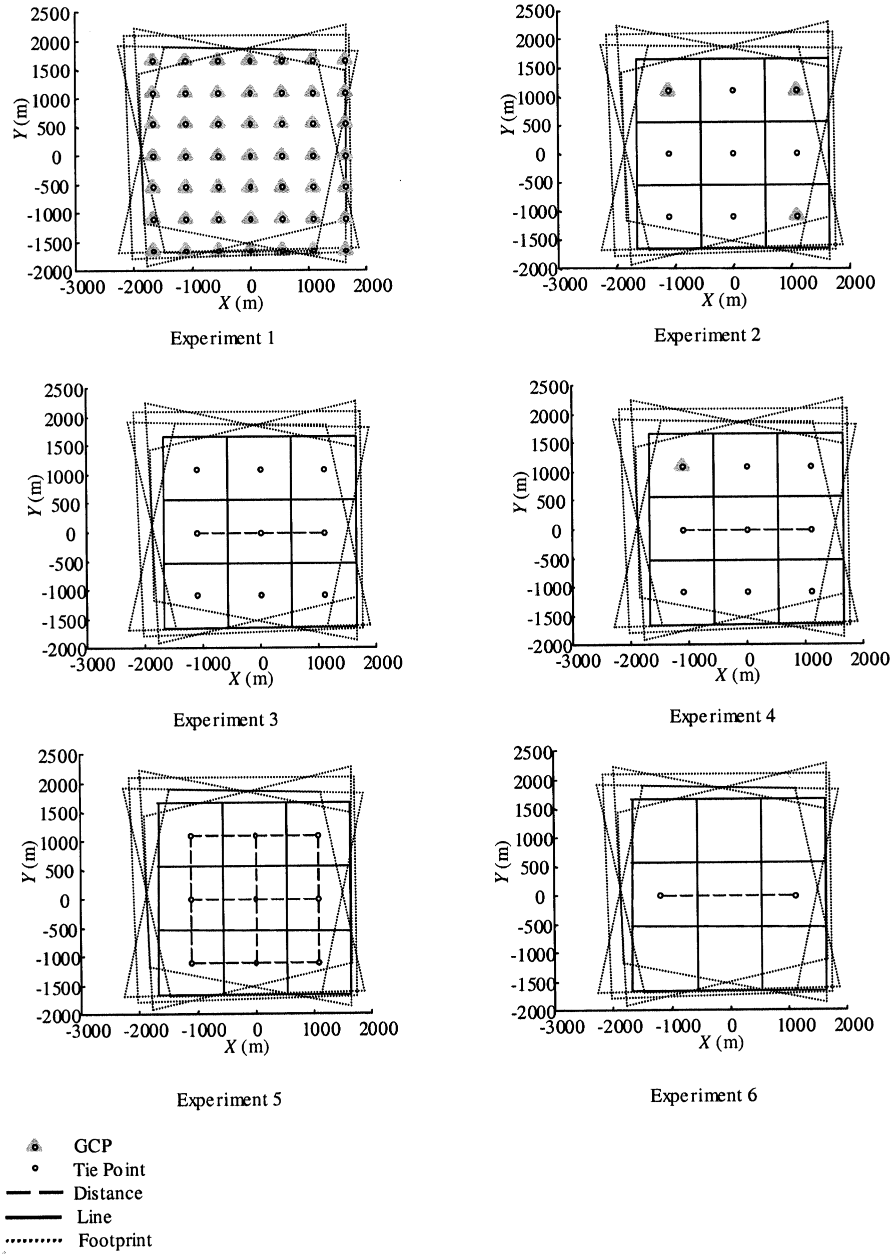

FIG. 4. Configuration diagrams for Experiments 1 to 6.

TABLE II. True and estimated values for IOPs (Experiments 1 to 6).

Parameter	True values	Experiment number 1	2	3	4	5	6
x_p (mm) $\times 10^{-3}$	0·0	1·55 ±6·85	−5·00 ±11·20	−3·53 ±11·43	−5·21 ±8·26	−7·36 ±9·87	−4·18 ±21·62
y_p (mm) $\times 10^{-3}$	0·0	7·22 ±7·95	0·147 ±10·64	2·12 ±11·05	−0·16 ±8·84	1·85 ±8·64	−8·37 ±20·84
c (mm)	150·0	149·984 ±0·036	150·023 ±0·027	150·023 ±0·027	150·003 ±0·011	150·018 ±0·027	149·970 ±0·034
K_1 (mm^{-2}) $\times 10^{-7}$	5·00	5·05 ±0·061	4·98 ±0·030	4·98 ±0·030	4·98 ±0·030	4·99 ±0·029	4·97 ±0·030
K_2 (mm^{-4}) $\times 10^{-13}$	0·0	−1·84 ±2·120	1·33 ±0·986	1·32 ±0·991	1·32 ±0·989	0·93 ±0·978	1·52 ±1·002
P_1 (mm^{-1}) $\times 10^{-6}$	5·00	5·02 ±0·095	5·06 ±0·068	5·06 ±0·068	5·07 ±0·066	5·06 ±0·068	5·09 ±0·071
P_2 (mm^{-1}) $\times 10^{-7}$	8·00	9·09 ±1·051	7·92 ±0·748	7·92 ±0·753	7·91 ±0·731	7·95 ±0·751	7·85 ±0·789
$A_1 \times 10^{-3}$	1·00	1·00 ±0·009	0·99 ±0·006	0·99 ±0·006	1·00 ±0·004	0·99 ±0·006	1·01 ±0·008
$A_2 \times 10^{-3}$	1·00	0·99 ±0·009	1·01 ±0·008	1·01 ±0·008	1·01 ±0·008	1·01 ±0·008	1·01 ±0·010

adjustment) with no control point. It can be seen that the distortion parameters have been correctly estimated.

The performance of the proposed approach can also be checked by investigating the quality of line fitting through the intermediate points in the image space before and after the calibration. The variance components of the line fitting adjustment in the first image of Experiment 5 before and after the calibration are shown in Fig. 6. As expected, the quality of line fitting is significantly improved after the calibration.

It has been reported in the literature that K_1 is the most significant distortion inherent in digital cameras (Seedahmed et al., 1998). By looking at Fig. 5(c), it can be seen that the use of straight-line constraints is much better for the recovery of this type of distortion. In general, this comment would still apply for the other IOPs in Figs. 5(a) to 5(e). It should be noticed that the influence of the affine transformation residuals in Fig. 5(f) on the measured image coordinates ($\sim 1\ \mu$m) does not exceed the noise level.

Real Data

The proposed algorithm for camera self-calibration using linear feature constraints has been tested using real data. The testfield used in this experiment is shown in Fig. 7. Six overlapping images have been captured covering the testfield. Signalised targets (crosses) as well as some natural targets have been used as tie points (a total of ten points). The datum for the bundle adjustment has been

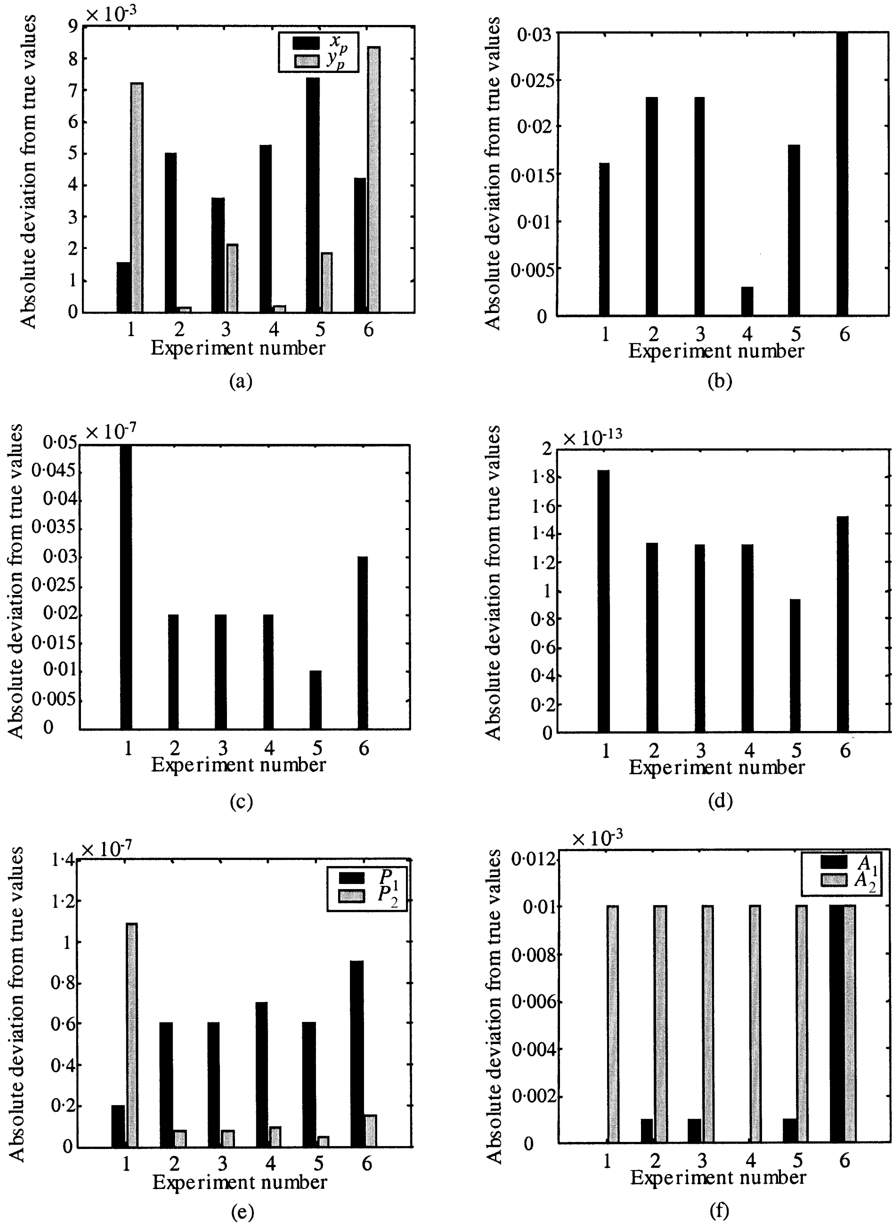

FIG. 5. Absolute deviations from the true values for the estimated IOPs: (a) x_p and y_p (mm); (b) c (mm); (c) K_1 (mm^{-2}); (d) K_2 (mm^{-4}); (e) P_1 and P_2 (mm^{-1}); (f) A_1 and A_2.

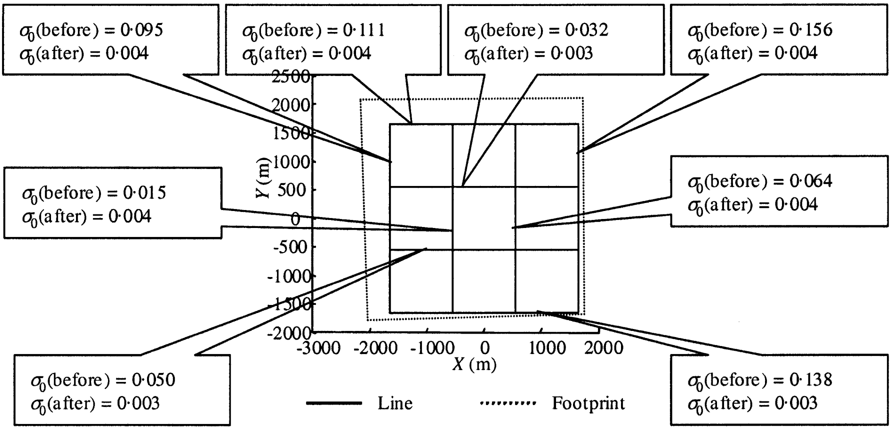

FIG. 6. Quality of line fitting before and after calibration for synthetic data.

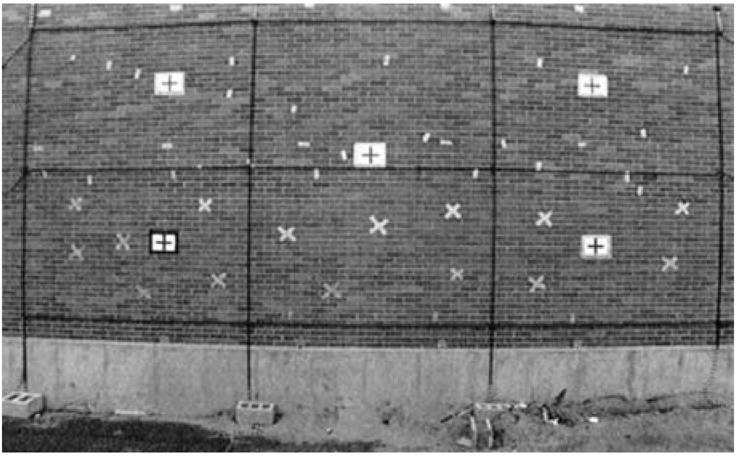

FIG. 7. Sample of input imagery used for bundle adjustment with self-calibration using straight-line constraints.

established by fixing the EOPs of one of the images to an arbitrary value, in addition to measuring the distances between the signalised targets (a total of nine distances). The estimated IOPs can be seen in Table III. In order to evaluate the quality of the results achieved, the images have been resampled after removing various distortions (Fig. 8). It can be seen that the straight lines have been correctly restored. A quantitative measure is developed using regression analysis applied to the measured intermediate points along the straight lines before and after calibration. These results can also be seen in Fig. 8. Once again, the computed variance components after the calibration are significantly better than those computed prior to the calibration process.

TABLE III. Estimated interior orientation parameters of the real data.

Parameter	*Value*
Estimated variance component (σ_0^2)	$5{\cdot}726 \times 10^{-3}$
x_p (mm)	$-0{\cdot}039$ ($\pm 0{\cdot}016$)
y_p (mm)	$-0{\cdot}197$ ($\pm 0{\cdot}021$)
c (mm)	$6{\cdot}733$ ($\pm 0{\cdot}021$)
K_1 (mm^{-2})	$-3{\cdot}989 \times 10^{-3}$ ($\pm 0{\cdot}041 \times 10^{-3}$)
K_2 (mm^{-4})	$0{\cdot}000$ ($\pm 0{\cdot}000$)
P_1 (mm^{-1})	$-3{\cdot}450 \times 10^{-4}$ ($\pm 1{\cdot}287 \times 10^{-4}$)
P_2 (mm^{-1})	$-5{\cdot}735 \times 10^{-4}$ ($\pm 0{\cdot}870 \times 10^{-4}$)
A_1	$-1{\cdot}964 \times 10^{-2}$ ($\pm 0{\cdot}017 \times 10^{-2}$)
A_2	$4{\cdot}277 \times 10^{-4}$ ($\pm 2{\cdot}677 \times 10^{-4}$)

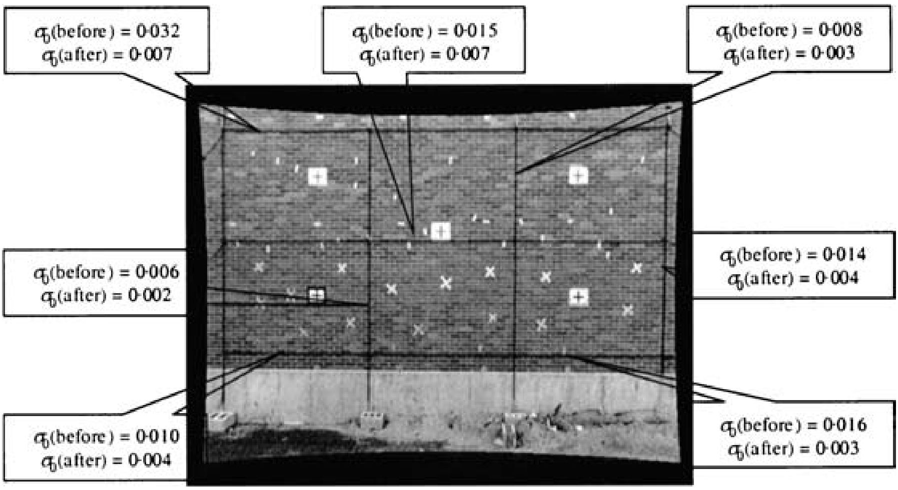

FIG. 8. Resampled version of image shown in Fig. 7 based on the estimated parameters from the self-calibration process.

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

In this paper, a new approach for performing bundle adjustment with self-calibration using straight lines has been presented. Two points, which are monoscopically measured in any of the images, represent straight lines in the object space. However, the image lines are defined by monoscopically measuring intermediate points along those lines. The perspective relationship between image- and object-space lines is incorporated in a mathematical constraint. The underlying principle in this constraint is that the vector from the perspective centre to an image point on a straight-line feature lies on the plane defined by the perspective centre and the two object points defining the straight line. Using straight lines for camera calibration is based on the principle that straight lines in the object space will be imaged by a frame camera as straight lines in the absence of distortions. Deviations from straightness in the image space are used to derive values for the various distortions. Experiments with synthetic and real data have proved the feasibility of this algorithm. Linear features eliminate the requirement for a testfield with a large number of ground control points for calibration. Establishing and maintaining such a testfield is an expensive and specialised process. The algorithm proposed is easy to incorporate in bundle adjustment procedures. It should be noted that this approach could be used for imagery captured by a line scanner. In this case, deviations from straightness in the imagery are caused by distortions as well as perturbations associated with platform movement during the scene capture.

The proposed calibration procedure is only applicable to images containing a group of straight lines. Establishing testfields with straight lines for close range applications in a controlled environment is an easy and straightforward process. Moreover, close range imagery of man-made scenes is rich in straight lines. For aerial imagery, special attention should be paid to ensuring the straightness of linear features prior to their use in the calibration process.

Future work will concentrate on automatic identification and point measurement along the lines in the image space. Creating an automatic procedure for camera calibration will enable users of digital imagery who are not photogrammetrists to deliver products with high quality. The inclusion of other primitives such as conic sections will also be investigated. Finally, the authors intend to extend this approach to imagery captured by line scanners.

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Résumé

Le développement de l'imagerie numérique a fourni l'occasion de recourir davantage aux détails des objets, et non pas seulement aux points qui les constituent, dans toute application photogrammétrique. C'est ainsi que les objets présentent souvent des lignes droites qu'il était tentant, dans une recherche antérieure, d'introduire pour contraindre la compensation par faisceaux d'imageries photographiques. On présente dans cet article cette recherche où les lignes droites de l'espace objet sont utilisées dans une compensation par faisceaux avec auto-étalonnage. En l'absence de distorsions, la projection de lignes droites de l'espace objet dans l'espace image s'opère également sous forme de lignes droites. Tout écart à une droite sur l'image peut donc être attribué à toutes sortes de distorsions, comme la distorsion radiale ou celle due au décentrement de l'objectif. Avant d'utiliser ces lignes droites dans une compensation par faisceaux avec auto-étalonnage, il faut effectuer la représentation et la transformation perspective des lignes droites entre les espaces objet et image. Dans cette démarche, on considère les images des lignes droites comme constituées d'une suite de points jalonnant cette image, tandis que dans l'espace objet cette ligne droite n'est définie que par deux points seulement. La relation de perspective qui relie les droites des espaces objet et image est alors introduite comme contrainte mathématique. Le principe de base de cette contrainte est que le vecteur issu du centre perspectif vers un point image d'une ligne droite de l'objet appartient au plan défini par ce centre perspectif et les deux points retenus dans la définition de cette droite. On a incorporé cette contrainte dans un logiciel appliqué à la compensation par faisceaux avec auto-étalonnage prenant en compte les points ainsi que les éléments en ligne droite de l'objet. Des essais avec des données simulées puis réelles ont montré la faisabilité et l'efficacité de l'algorithme proposé.

Zusammenfassung

Durch die zunehmende Nutzung digitaler Bilder wurde die Möglichkeit geschaffen, neben Punkten auch Objektmerkmale in photogrammetrischen Anwendungen zu nutzen. Oftmals finden sich Geraden im Objektraum, und frühere Forschung hat sich darauf konzentriert, Linienbedingungen in die Bündelausgleichung für Flächenkameras zu entwickeln. In den hier vorgestellten Forschungen werden Geraden im Objektraum in einer Bündelausgleichung mit Selbstkalibrierung eingesetzt. Wenn keine Verzeichnungen vorliegen, werden bei einer perspektiven Abbildung Geraden im Objektraum in Geraden im Bildraum abgebildet. Jegliche Abweichung von einer Geraden im Bildraum kann mit verschiedenen Ursachen für Verzeichnung in Verbindung gebracht werden, wie zum Beispiel radiale *oder asymmetrische Objektivverzeichnung. Bevor Geraden in die Bündelausgleichung mit Selbstkalibrierung eingehen können, sollte die Repräsentation und die perspektive Transformation der Geraden zwischen Bild- und Objektraum geklärt werden. In dieser Untersuchung werden die Abbildungen der Geraden als eine Sequenz von Punkten entlang einer Bildlinie dargestellt. Zwei Punkte entlang der Objektgeraden werden genutzt, um diese darzustellen. Die perspektive Beziehung zwischen Bild- und Objektgeraden wird mit Hilfe einer mathematischen Bedingung formuliert. Das Prinzip, das dieser Beziehung zugrunde liegt, geht davon aus, dass ein Vektor vom Projektionszentrum zu einem Bildpunkt auf einer Geraden auf einer Ebene liegt, die durch das Projektionszentrum und die zwei Objektpunkte, die die Gerade definieren, bestimmt wird. Diese Bedingung wurde in ein Anwendungsprogramm zur Bündelausgleichung mit Selbstkalibrierung eingebaut, das sowohl Punkt- als auch Geradenmerkmale verarbeiten kann. Experimente mit simulierten und echten Datensätzen belegen die Anwendbarkeit und die Effizienz des vorgeschlagenen Algorithmus.*