

# A New Approach of Combined Block Adjustment Using GPS-Satellite Constellation

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## ABSTRACT

The ambiguities of kinematic GPS positioning based on carrier-phase measurements in an aircraft are often not resolved correctly. This causes systematic errors in the GPS positions depending on time and location. In practical applications of combined block adjustment with projection center coordinates determined by GPS commonly shift and drift parameters for the fitting of the errors in the GPS-positions are introduced. However, modeling remaining GPS systematics by linear regression assumes a linear behavior of the errors. Non linear effects have been detected for poor GPS satellite geometry. A new approach of combined block adjustment is presented, which uses GPS satellite constellations and strictly obeys the functional GPS model. Parameters can be estimated for a complete block. The new approach is discussed and preliminary results are given.

The shift and drift compensation in the traditional method has to be done individually for every flight strip. By this reason crossing flight strips or lines of vertical control points are required for blocks with a sidelap less than 50%. With the new method one correction for every continuous used satellite is required. That means, if some satellites have not had cycle slips during the whole flight period, these satellites are connecting the flight strips and crossing flight strips or additional vertical control points don't have to be used.

## 1. INTRODUCTION

The integration of the Global Positioning System (GPS) and bundle block adjustment for the reduction of the number of required control points has become an operational technique. For large scale mapping and also for height determination the GPS positioning has to be determined with a precision at the one decimeter level or even below and therefore must be based on carrier phases.

The capabilities of ambiguity resolution techniques for kinematic GPS positioning are steadily improving and have already achieved a high reliability level. The accuracy of kinematic GPS for dynamic applications depends on the distance to the reference GPS station and the used observable. In marine applications, even over distances of up to 60 km accuracy's below the one decimeter level are reported using carrier phase measurements (Seeber 1995). However, for highly precise airborne applications the ambiguities and cycle slips cannot always be solved or, sometimes, are not correctly resolved. False ambiguity resolution introduces systematic errors into the GPS positions.

The general approach in a combined bundle adjustment uses linear regression models or even polynoms to adjust the systematic distorted GPS positions to the projection centers. This has to be done in the block adjustment individually for every flight strip because the GPS-position errors are not only a function of the time, they are depending also upon the location. In addition in the turn around from one flight strip to the next usually the connection to some satellites are lost, causing a change in the systematic errors

The ambiguity errors are mainly causing deviations in the GPS-positions linear depending upon the time and location. However, non-linear effects have been found for poor satellite constellations. The remaining errors of the GPS position raised the ground height accuracy derived by a combined block adjustment and independent check points by a factor of 2 (Schmitz 1995). Such errors cannot be detected in an operational block without independent check points.

A new approach of combined adjustment is presented, which uses the actual satellite constellation to determine GPS position corrections. Actually, the approach can be described as an improvement of the ambiguity terms using the independent position information from the bundle adjustment. Preliminary results are given.

## 2. DATA SETS

In 1993 an extensive test of GPS supported block adjustment (photo flight mission 'Vechtel') has been performed in cooperation between the state survey of the German federal country Lower Saxony and the University of Hannover (Jacobsen 1994, Jacobsen 1996). From the operational production of photo flights for height determination additional data sets are available from the state survey. The investigations in this paper are based on the data set 'Vechtel' (1993) and 'Gross Oesingen' (1994).

The blocks were imaged by a Zeiss RMK TOP 15 with an image scale of approximately 1:8000. The strips were flown in east-west direction with a sidelap of 60 % and stabilized in the case of 'Vechtel' by 5 strips in North-South direction. A GPS photo flight system has been used for navigation. However, due to the 60% sidelap also 'Oesingen' can adequately be used for a combined

adjustment. The block areas were covered by equally distributed and targeted control points, which allows independent control with a number of check points. The accuracy of the control points in the national network is +/- 2 cm for all coordinate components.

The aircraft and the reference stations in the flight area were equipped with geodetic Trimble 4000 SSE GPS receivers. In addition, permanent reference stations (Trimble 4000 SSE) operated at different distances from the project areas. The elevation mask of the moving receiver in the aircraft was 5 degree and the recording interval was set to 0.5 sec.

Unfortunately, in both photo flights only a L1-only processing was possible due to a too much corrupted L2 signal for 'Vechtel' and a receiver handling error for 'Oesingen'. Generally, the coverage of the satellite constellation with always more than 5 satellites during a flight strip is fair for 'Vechtel' in 1993 as the constellation of 6-8 satellites is good for 'Oesingen' in 1994. The GPS geometry described by the position dilution of precision (PDOP) is better than 5, except for individual strips in 'Vechtel' with a PDOP up to 30.

The bundle block adjustment has been performed with the program system BLUH developed at the University of Hannover. For GPS post-processing the GEONAP software package was used, which is capable to handle and combine a number of ambiguity resolution techniques. It is based on the parameter estimation approach and uses undifferenced GPS observations.

### 3. DIFFICULTIES OF COMBINED ADJUSTMENT

Associated with GPS supported bundle block adjustment are some general problems, which will not be discussed here in detail. Assuming a solid and stable connection of the camera to the body of the aircraft, ideally three conditions have to be fulfilled:

- identical time tag of GPS and time of exposure
- identical reference point (i.e. no eccentricity of GPS antenna)
- identical geodetic datum.

Identical time scales can be achieved by using a time tag generated by the GPS receiver to trigger the photo imaging. Having no actual time synchronization a sufficient interpolation using linear or polynomial fitting is applied to the GPS positions.

The eccentricity is preferable determined with a conventional survey before or after the flight. Using the precise exterior orientation angles derived from the bundle block adjustment the eccentricity can be adequately considered if the camera is not moved in relation to the aircraft. Otherwise the camera rotations in relation to the aircraft have to be recorded.

In most cases the national network is of interest for the photogrammetric results. That requires a transformation of the WGS84 GPS positions to the local datum. The geoid undulations are generally not considered. Therefore, remaining datum differences in the datum are estimated in the GPS supported block adjustment.

### 3.1 Changes of Satellite Geometry

The GPS position accuracy  $\sigma_X$  can be described using the PDOP and the range error  $\sigma_r$  (Seeber 1993)

$$\sigma_X = \sigma_r \cdot PDOP. \quad (1)$$

PDOP numbers refer to a single station and do not reflect precisely DGPS, but they can be used for a general indication of the geometric DGPS conditions (Schmitz 1995). On the one hand, any changes of the satellite constellation cause a change in geometry and hence in the position estimation. On the other hand, high PDOP numbers correspond to less accurate positions.

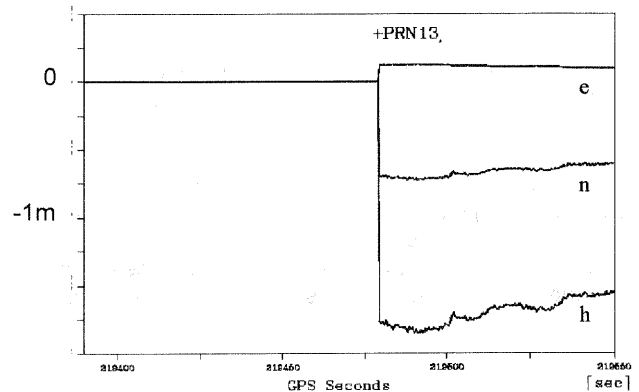


Figure 1: Strip 22 of photo flight Vechtel: Difference in northing, easting and height for kinematic GPS station with and without PRN13

An example of the effect of geometric changes is given in Figure 1. PRN13 is not permanently tracked for some strips of the photo flight 'Vechtel'. After fixing all possible ambiguities using the 20-30 km remote reference station 'Meppen', all remaining ones were forced to the nearest integer and the GPS positions with and without PRN13 were compared. Large offsets of up to 1.75 m can be seen, when satellite PRN13 is considered in the middle of the strip. The large magnitude is partially due to a high PDOP of 20-25. However, satellites should be permanently visible during a strip to avoid any discontinuities in the GPS positions in the case of the traditional method of combined block adjustment (Schmitz 1995).

### 3.2 GPS Systematics and Assumption of Linearity

False ambiguity resolution introduces systematic errors into the GPS position. In most cases, the systematics show a linear drift behavior of the coordinate components. In Figure 2 the effects of three different magnitudes of coordinate differences from small variations up to one decimeter variations are visible. An absolute offset of the position due to missing observations for PRN13 (geometry change) is again visible at the end of the strip.

With the assumption of linear systematics in the GPS positions, a linear regression is generally applied using shift (systematic offset) and drift parameters (time dependent behavior) to adjust the GPS to the projection center derived from block adjustment.

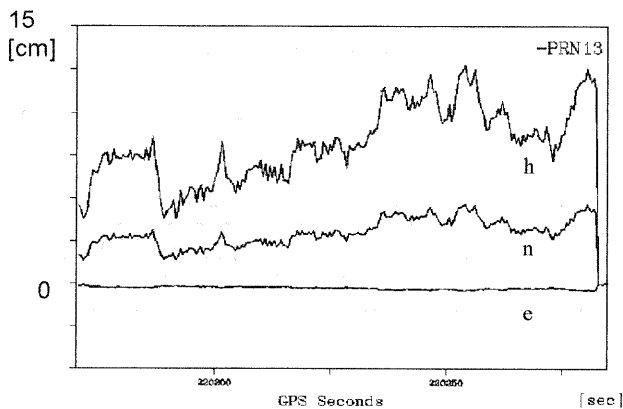


Figure 2: Strip 23 of photo flight Vechtel: Difference in northing, easting and height for kinematic GPS station with and without PRN13

The linearity of GPS systematics has been investigated using data from 'Vechtel'. Generally, systematic effects and discontinuities in GPS trajectories are not visible in the absolute positions, because they cannot be separated from the actual aircraft flight behavior (turbulence, etc.). An adequate tool for detailed investigation is the comparison of different GPS evaluations (e.g. independent stations) or artificially manipulated data sets (e.g. falsifying ambiguities). Then systematic errors magnify and show up.

To investigate the GPS systematics the ambiguity of one satellite was falsified by 1 cycle and compared to the initial solution. After fixing as much ambiguities as possible, the remaining were forced to integers and the GPS positions were estimated individually for each strip without any cycle slip or change of satellite constellation using reference station 'Meppen'.

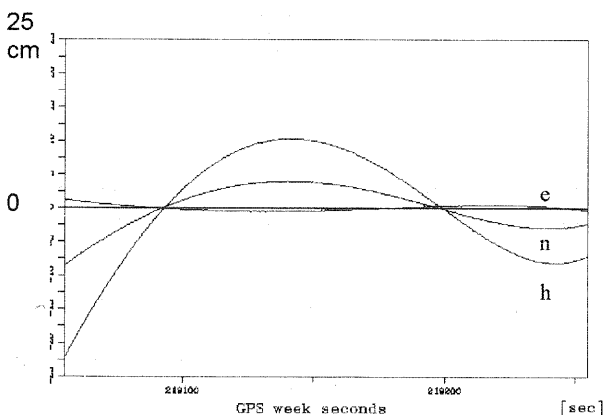


Figure 3: Strip 21: Residuals after linear regression of coordinate differences (PRN26 + 1 cycle)

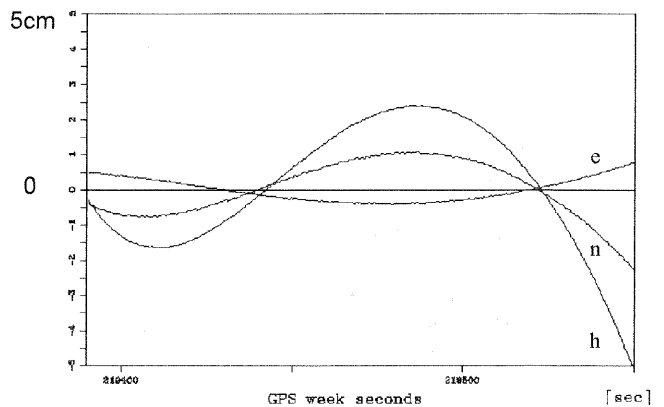


Figure 4: Strip 22: Residuals after line regression of coordinate differences (PRN26 + 1 cycle)

Applying a linear regression to the coordinate differences between falsified and best solution gives an idea of the not modeled behavior in the GPS supported block adjustment. Figure 3 and Figure 4 show the residuals from a linear regression in northing, easting and height. Significant deviations from linearity can be observed, which amounts to some decimeters. They correspond to remaining errors, which will not be modeled in the block adjustment and therefore add uncertainties to the GPS positions.

#### 4. NEW APPROACH USING GPS SATELLITE GEOMETRY

Comparisons of GPS kinematic trajectories with the projection centers derived from a conventional block adjustment show agreement at about 5 cm (Seeber et al. 1995). Obviously, the bundle block adjustment, even with a reduced number of control points, can serve as a constraint in the determination of ambiguities in the combined adjustment.

Instead of modeling the systematic errors introduced by false ambiguity resolution by a linear regression, which does not suit the GPS model, a new approach to the combined adjustment is proposed. Actual ambiguity terms are improved in the combined bundle adjustment using geometric GPS information and information on ambiguities from the GPS processing.

The GPS and the combined block adjustment are performed in the general way of intermediate adjustments. The observation equation for the GPS coordinates in the combined adjustment reads

$$l = A\bar{x} + v. \quad (2)$$

It can be extended by an additional ambiguity term, which will be estimated in the combined adjustment

$$l + \lambda \cdot N = A\bar{x} + v. \quad (3)$$

This requires, that the design matrix contains the complete geometric information from the GPS adjustment. Considering the normal equation

$$\bar{x} = Qn = (A^T P A)^{-1} (A^T P l), \quad (4)$$

we arrive at

$$\bar{x} = (A^T P A)^{-1} (A^T P (l + \lambda \cdot N)). \quad (5)$$

Using a global term  $\Delta N = \lambda \cdot N$  for the improvement in direction to the satellites one yields

$$\bar{x} = Q A^T P l + Q A^T P (\Delta N). \quad (6)$$

The design matrix contains the direction cosines of each observed satellite (geometric information) and reflects ambiguities and cycle slips.

To keep the number of additional ambiguity terms small, only those, which could not be fixed reliably in the GPS processing are improved. Therefore, coordinates with 'improved' ambiguities can be computed from a 'forced' coordinate estimation done by the GPS processing software and improvements of the coordinate estimates ' $\Delta N$ ' from the combined bundle adjustment:

$$\bar{x}_{improved} = \bar{x}_{forced} + \bar{x}_{\Delta N} \quad (7)$$

For this GPS error model - some ambiguities already solved, some still not reliably fixed, but forced to integers - the corresponding design matrix is evaluated to be incorporated into a combined bundle adjustment. The new GPS model for remaining systematic GPS errors can be handled by the software packages GEONAP and BLUH.

## 5. EMPIRICAL RESULTS

The new approach was applied to the data set 'Oesingen'. In a first processing of the GPS data all ambiguities and cycle slips are estimated and fixed, if the ambiguity resolution algorithms have succeeded using stochastic criterion's. One satellite was badly corrupted by cycle slips and rejected completely from the GPS processing. In total 18 ambiguities/cycle slips are present for the complete block. However, 4 ambiguities could be reliably fixed using the stochastic properties of the GEONAP processing software.

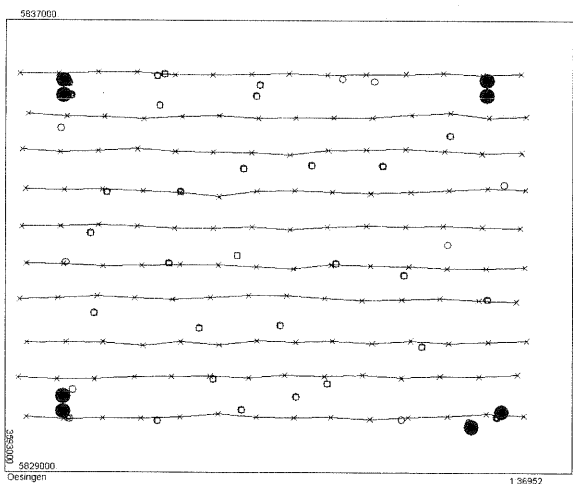


Figure 5: block Oesingen with control and check points 140 photos, image scale 1 : 8000, f=153mm

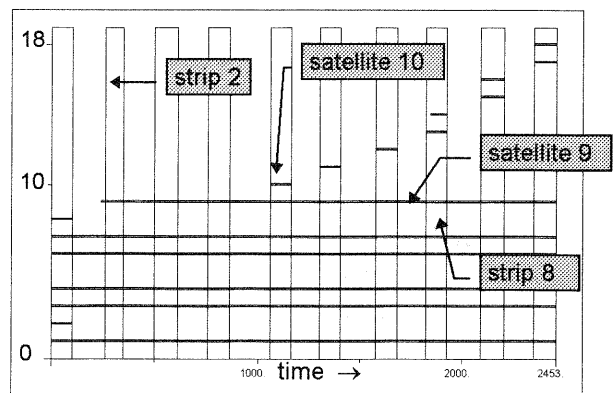


Figure 6: used satellites block Oesingen  
horizontal lines: continuously used satellites

Figure 6 gives an overview about the used satellites in the block Oesingen. 10 flight strips have been used. The satellites 1, 3, 4, 6 and 7 could be used without cycle slip over the whole flight time, satellite 9 was not used in the first flight strip. The other satellites have been used only in one flight strip. The satellite number in figure 6 does not correspond to the numbers of the GPS-satellites, they are only internal numbers. After a cycle slip the internal satellite number is changed. The satellite 14 is not used in the whole strip 8. In the traditional method of combined block adjustment this can cause an inhomogeneity of the solution, with the new approach also this satellite can be used.

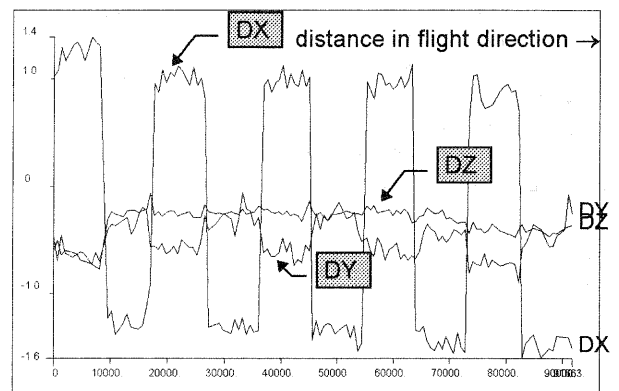


Figure 7: Differences GPS-positions against projection centers determined by reference adjustment with BLUH

The differences between the interpolated GPS-positions and the projection center coordinates determined by bundle block adjustment without GPS-data are shown in figure 7. The offset of the GPS-antenna was not recorded, but it can be seen in figure 7 as -1.2m in X and 0.15m in Y. In addition the typical changes depending upon the flight strips are obvious. A fitting of the differences individually for every flight strip by constant values (shift) yields to  $RMSX = \pm 9.1\text{cm}$ ,  $RMSY = \pm 9.2\text{cm}$  and  $RMSZ = \pm 4.3\text{cm}$ . A linear regression (drift) reduces the differences only to  $RMSX = \pm 9.1\text{cm}$ ,  $RMSY = \pm 9.1\text{cm}$  and  $RMSZ = \pm 3.7\text{cm}$ . The differences are not only caused by the GPS-positioning, the root mean square of the projection center accuracy determined by block adjustment is  $RMSX_0 = \pm 5.2\text{cm}$ ,  $RMSY_0 = \pm 5.7\text{cm}$  and  $RMSZ_0 = \pm 2.8\text{cm}$ .

A combined block adjustment was made with all photos (sidelap  $p=60\%$ ) and 8 control points (2 in every block corner - see figure 5) and also with a reduced set of 70 photos with a sidelap of 20%.

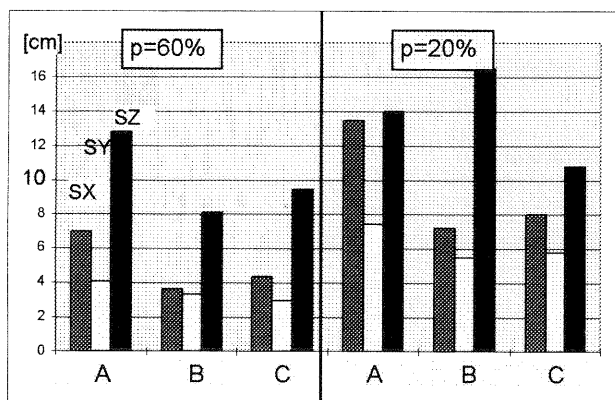


Figure 8: results of the bundle block adjustment  
 A: without GPS  
 B: combined adjustment old method  
 C: combined adjustment new approach

Figure 8 shows the results of the bundle block adjustments of the block Oesingen with the program system BLUH at independent check points. On the left hand side are the results of the adjustment with all 140 photos and a sidelap of 60%. The stable block configuration do not show an advantage of the new method of combined block adjustment, the results of the traditional method is in X and Y with  $\pm 3.5\text{cm}$  nearly the same like with the new method of  $\pm 3.6\text{cm}$ . In the height the old method shows  $\pm 8.1\text{cm}$  and the new method  $\pm 9.5\text{cm}$ .

The results of the block adjustment with only 70 photos and a sidelap of 20% can be seen on the right hand side. The horizontal accuracy is approximately the same for both methods, but this has been expected in advance - in general in the block adjustment the horizontal geometry is stable also without GPS and without additional vertical control points. The vertical component is different. A block with just 20% sidelap and no crossing flight strips is not stable in the height. Also the traditional method of combined block adjustment will not solve the problem because every flight strip is shifted individually, but here the advantage of the new approach can be seen - with the old method a vertical accuracy of only  $SZ=\pm 16.5\text{cm}$  is reached, by the new method the vertical component is improved to  $SZ=\pm 10.8\text{cm}$ .

Beside the general additional parameters for the self calibration one unknown for every satellite has been introduced. In addition unknowns for the datum shift (3 components) and for the determination of errors of the antenna offset are required. If all used strips have been flown in the same direction, it is not possible to introduce unknowns for the datum shift and for the antenna offset.

## 6. BENEFITS FROM THE NEW APPROACH

The general approach of shift and drift parameters uses independent GPS positions for each strip. The new approach using satellite geometry considers also the geometric relationship between strips. It is possible to

estimate one set of additional parameters for the complete block obeying strictly the functional GPS model. Additionally, no special investigations on satellite geometry changes during a strip are required for the GPS processing.

An additional problem is the distance dependence of precise GPS. Over large distances the block adjustment can constrain the ambiguity resolution. Detailed investigations on this topic are necessary.

The number of additional parameters is generally reduced, if not during every curve flight all satellite signals are lost.

## 7. CONCLUSION

For a number of applications in photogrammetry, which primarily depend on the scale, only GPS positions accurate to some decimeter or less are sufficient. For high precise demands, the correct ambiguity resolution is required to compute GPS positions at the 5 cm level.

A new approach of combined block adjustment has been presented, which uses strictly the functional GPS model to estimate remaining systematic errors in the GPS positions caused by false ambiguity resolution.

Even the re-substitution of the improved ambiguity terms is feasible, however, it is not of much interest as the GPS processing techniques improve.

The first test of a block with a limited size always shows the advantage of the new method. Crossing flight strips can be avoided even in blocks with just 4 control points and 20% sidelap.

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