A Photogrammetric Solution for Projective Reconstruction

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ABSTRACT

This article proposes a novel solution for projective reconstruction for computer vision with non-metric camera. After a brief introduction of projective transformation as well as projective invariant and coordinates, the fundamental matrix is expressed as a production of two matrices, the projective base line matrix and the projective rotation matrix. This derivation is based on photogrammetric concepts and thus close similarity is found to the decomposition of the essential matrix for metric camera. A projective coefficient, which is proven to be the cross ratio of lengths of two conjugate projective rays, is therefore derived to calculate the homogeneous coordinates and projective coordinates (cross ratio of volume elements) of the projective model. In order to reconstruct the object from its projective model, as an extension to the well-known 2-D direct linear transformation (2-D DLT) solution for metric camera in conventional photogrammetry, a 3-D DLT solution is proposed. The reconstruction is linearly completed with minimum five conjugate known object points. Test results and analyses verify the solution and methodology.

Keywords: computer vision; fundamental matrix; photogrammetry; projective invariant; projective reconstruction

1. INTRODUCTION

Despite the difference in application fields, photogrammetry and computer vision have the same theoretic background and many joint efforts have been made in past years [Hartley, et al, 1993]. As a convention, photogrammetry uses metric camera for its topographic applications where interior orientation of the camera is often calibrated and known. On the contrary, non-metric camera is the main device for image acquisition and thus becomes an important topic in computer vision. As a subject with common applications to computer vision, close-range photogrammetry has developed and used the direct linear transformation algorithm (DLT) to process non-metric images in the last 30 years [Abdel-Aziz, et al, 1971]. This algorithm is based on the collinearity in single image. Although comprehensive theory has been developed for stereo pair of metric camera in photogrammerty, there lacks such a similar methodology dealing with a stereo pair taken from non-metric camera - the DLT remains the major processing algorithm based on single photo calculation. Recent literatures in computer vision have been focused on object reconstruction from a non-metric stereo pair. It has been shown that the recovery is up to a projective transformation if the interior orientation is not known. Various algorithms for projective recovery have been discussed and developed in recent years in computer vision field [Faugeras, 1992; Hartley, 1992; Rothewell, et al, 1997].

This article addresses the projective reconstruction from photogrammetric perspective view. The objective is to develop a solution and methodology for non-metric camera similar to metric camera. This can be considered as a generalization of conventional photogrammetric solution where the metric camera is the major concern. In section 2, the general concept of projective geometry is briefly introduced where cross ratio of volume elements is shown as projective invariant and therefore defined as projective coordinate. Section 3 introduces the fundamental matrix by using photogrammetric derivations. It is shown that the fundamental matrix can be expressed, up to a constant, as a product of the projective base line matrix (a skew symmetric matrix) and the projective rotation matrix. Derivatives from the fundamental matrix are the ratios of the base line components, six relations for the elements of the projective rotation matrix and the cross ratio of lengths of two conjugate projective rays. These derivatives will be used for the reconstruction of the projective model of the object. Section 4 first shows that the projective model can be defined with its homogenous coordinates which in turn are calculated with the derivatives from the fundamental matrix. Similar to the 2-D DLT solution for single photograph, a 3-D DLT solution is designed which completes the projective transformation for the projective model to its object space. In this way, the object is fully reconstructed by first forming its projective model with the derivatives of the fundamental matrix, and then followed by a 3-D DLT operation which conducts the projective transformation to the projective model. Tests given in Section 5 verify the development and the proposed solution. Results show that 3-D DLT algorithm can reach compatible results with the 2-D DLT algorithm where the former needs five conjugate known object points instead of six known object points on each photograph of a stereo pair. Final concluding remarks are summarized in Section 6.

2. PROJECTIVE TRANSFORMATION, INVARIANTS AND COORDINATES

1. Projective transformation

The mathematics of photography is based on projective geometry. A general projective transformation was introduced by [Brill et al, 1983]. Its 3-D expression can be written as

$$\begin{aligned} U' &= \frac{a_{11}U + a_{12}V + a_{13}W + a_{14}}{a_{41}U + a_{42}V + a_{43}W + a_{44}} \\ V' &= \frac{a_{21}U + a_{22}V + a_{23}W + a_{24}}{a_{41}U + a_{42}V + a_{43}W + a_{44}} \\ W' &= \frac{a_{31}U + a_{32}V + a_{33}W + a_{34}}{a_{41}U + a_{42}V + a_{43}W + a_{44}} \end{aligned} \tag{1}$$

This equation can also be expressed with homogeneous coordinates as follows

$$t\,\bar{\mathbf{u}}' = \mathbf{A}\,\bar{\mathbf{u}} \tag{2}$$

where

$$\begin{aligned} \bar{\mathbf{u}} &= (U\,V\,W\,1)^T \\ \bar{\mathbf{u}}' &= (U'V'W'1)^T \end{aligned} \tag{3}$$

are homogeneous coordinates of an object point before and after the transformation. As a convention in photogrammetry, $\bar{\mathbf{u}}$ and $\bar{\mathbf{u}}'$ are called object coordinate and model coordinate, respectively. $\mathbf{A} = \{a_{ij}\}$ is the 4x4 projective transformation matrix. $t \neq 0$ is a constant factor relevant to that object point.

2. Projective invariants

It will be shown that the cross ratio of volume elements is invariant under 3-D projective transformation. This can be understood as a generalization of the cross ratio of length segments in 1-D [Duda et al, 1973] and the cross ratio of area elements in 2-D. The volume element Δ_{ijkl} is calculated with following determinant

$$\Delta_{ijkl} = \begin{vmatrix} U_i & U_j & U_k & U_l \\ V_i & V_j & V_k & V_l \\ W_i & W_j & W_k & W_l \\ 1 & 1 & 1 & 1 \end{vmatrix} = (\bar{\mathbf{u}}_i\,\bar{\mathbf{u}}_j\,\bar{\mathbf{u}}_k\,\bar{\mathbf{u}}_l) \tag{4}$$

The cross ratio of volume elements is then defined with the determinants defined by five given points *1,2...5* and any point *i* as follows

$$C(1,2,3,4,5,i) = \frac{\Delta_{1234}}{\Delta_{1235}} \frac{\Delta_{125i}}{\Delta_{124i}} \tag{5}$$

It can be shown that such defined cross ratio is invariant under projective transformation [Brill et al, 1983; Barrett et al,1994]. It should be pointed out that the five points *1,2...5* in the above definition form a basis of a projective transformation. This basis can be globally or locally chosen, depending on application requirements.

3. Projective coordinates

Projective coordinate was introduced by [Duda, et al, 1973], which intends to describe an object with invariant quantities under projective transformation. Due to the invariance of the projective coordinates, they can be used for object reconstruction and recognition. By permuting the indexes in Eq.(5), following coordinates are chosen as projective coordinates for object point *i* relative to the basis *1,2...5*

$$\begin{aligned} C_1(i) &= \frac{\Delta_{1234}}{\Delta_{1235}} \frac{\Delta_{125i}}{\Delta_{124i}} \\ C_2(i) &= \frac{\Delta_{1324}}{\Delta_{1325}} \frac{\Delta_{135i}}{\Delta_{134i}} \\ C_3(i) &= \frac{\Delta_{2314}}{\Delta_{2315}} \frac{\Delta_{235i}}{\Delta_{234i}} \end{aligned} \tag{6}$$

Many works have been done on the computation of the projective coordinates and invariants [Barrett et al, 1991; Barrett et al, 1995; Hartley, 1994]. In the following sections, a photogrammetric solution will be developed.

3. FUNDAMENTAL MATRIX AND ITS DECOMPOSITION

1. Fundamental matrix

The fundamental matrix will be introduced by using the concept of the essential matrix. The essential matrix is related with metric camera with known interior elements. The coplanarity equation is written as [Longuet-Higgins,1981; Thompson, 1968]

$$\mathbf{x}_1^T \mathbf{E} \mathbf{x}_2 = 0 \tag{7}$$

where

$$\begin{aligned} \mathbf{x}_1 &= (x_1\, y_1\, -f_1)^T \\ \mathbf{x}_2 &= (x_2\, y_2\, -f_2)^T \end{aligned} \tag{8}$$

are image coordinates in image spaces, $(x_1\ y_1), (x_2\ y_2)$ are image coordinates on image planes, f_1, f_2 are principal lengths, respectively for left and right images. Matrix $\mathbf{E}$ is called essential matrix [Longuet-Higgins,1981; Thompson, 1968] and can be expressed as

$$\mathbf{E} = \mathbf{B}\,\mathbf{R} \tag{9}$$

where $\mathbf{B}$ is the screw symmetric matrix composed by base line components

$$\mathbf{B} = \begin{bmatrix} 0 & -B_Z & B_Y \\ B_Z & 0 & -B_X \\ -B_Y & B_X & 0 \end{bmatrix} \tag{10}$$

R is the orthogonal rotation matrix between left and right image spaces.

For non-metric camera, following notations are used

$$\begin{aligned} \bar{\mathbf{x}}_1 &= (\bar{x}_1\ \bar{y}_1\ 1)^T \\ \bar{\mathbf{x}}_2 &= (\bar{x}_2\ \bar{y}_2\ 1)^T \end{aligned} \tag{11}$$

where $\bar{\mathbf{x}}_1$ and $\bar{\mathbf{x}}_2$ are homogeneous coordinates of image points. Their relationship with Cartesian coordinates is

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{A}_1\bar{\mathbf{x}}_1 \\ \mathbf{x}_2 &= \mathbf{A}_2\bar{\mathbf{x}}_2 \end{aligned} \tag{12}$$

where

$$\mathbf{A}_1 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & -f_1 \end{bmatrix}_1 \qquad \mathbf{A}_2 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & -f_1 \end{bmatrix}_2 \tag{13}$$

are interior orientation matrices for left and right images respectively.

Substituting Eq.(12) into Eq.(7) will yield

$$\bar{\mathbf{x}}_1^T\,\bar{\mathbf{E}}\,\bar{\mathbf{x}}_2 = 0 \tag{14}$$

where

$$\bar{\mathbf{E}} = \mathbf{A}_1^T\mathbf{E}\mathbf{A}_2 = \mathbf{A}_1^T\mathbf{B}\,\mathbf{R}\,\mathbf{A}_2 \tag{15}$$

Matrix $\bar{\mathbf{E}}$ is called the fundamental matrix for this stereo pair [Faugeras, 1992]. The details about above derivation can be found in [Shan, 1996; 1997]. Eq.(14) is used to determine the fundamental matrix. It is a linear homogeneous equation with eight parameters, among which only seven are independent. That is because the fundamental matrix is rank deficient, namely

$$|\bar{\mathbf{E}}| = 0 \tag{16}$$

This equation will be used as a constraint to stabilize the solution of the fundamental matrix [Barakat et al, 1997]

2. Decomposition of the fundamental matrix

In the following section, decomposition is conducted to the fundamental matrix. A set of formulas is obtained which are similar to the ones in metric camera photogrammetry.

For metric camera, the model coordinates of a point are expressed as

$$\mathbf{p} = \lambda_1\mathbf{x}_1 = \lambda_2\mathbf{R}\,\mathbf{x}_2 + \mathbf{b} \tag{17}$$

where $\mathbf{b} = (B_X\ B_Y\ B_Z)^T$ is the base line vector, $\mathbf{p}$ is the model coordinate vector of a point, λ_i *(i=1,2)* is length ratio of projective rays, namely the length ratio of vector $\mathbf{p}$ and vector $\mathbf{x}_1$, and vector $\mathbf{p}-\mathbf{b}$ and $\mathbf{x}_2$. Substituting Eq.(12) into Eq.(17) will give the relationship for quantities of non-metric camera

$$\overline{\mathbf{p}} = \bar{\lambda}_1 \overline{\mathbf{x}}_1 = \bar{\lambda}_2 \overline{\mathbf{R}}\,\overline{\mathbf{x}}_2 + \overline{\mathbf{b}} \tag{18}$$

where

$\bar{\lambda}_i = \lambda_i$ *(i=1,2)*

$$\overline{\mathbf{p}} = \mathbf{A}_1^{-1}\mathbf{p} \tag{19}$$

is defined as model coordinates for non-metric camera with unknown interior parameters;

$$\overline{\mathbf{b}} = \mathbf{A}_1^{-1}\mathbf{b} = (\overline{B}_X\ \overline{B}_Y\ \overline{B}_Z)^T \tag{20}$$

is called the projective base line vector

$$\overline{\mathbf{R}} = \mathbf{A}_1^{-1}\mathbf{R}\,\mathbf{A}_2 \tag{21}$$

is called the projective rotation matrix. Those quantities are thus defined because comparing Eq.(18) with Eq.(17) will show that that model coordinates for non metric camera take similar forms as for metric camera. These definitions will be further reasoned in the following discussion.

To decompose the fundamental matrix, Eq.(15) can be rewritten as

$$\overline{\mathbf{E}} = (\mathbf{A}_1^T\mathbf{B}\mathbf{A}_1)(\mathbf{A}_1^{-1}\mathbf{R}\mathbf{A}_2) = \overline{\mathbf{B}}\,\overline{\mathbf{R}} \tag{22}$$

where

$$\overline{\mathbf{B}} = (\mathbf{A}_1^T\mathbf{B}\mathbf{A}_1) \tag{23}$$

As is shown in [Shan, 1996] $\overline{\mathbf{B}}$ is a skew symmetric matrix and its elements are proportional to the elements in vector $\overline{\mathbf{b}}$. Since this proportional constant can be taken into account in the calculation of the fundamental matrix, we can simply let

$$\overline{\mathbf{B}} = \begin{bmatrix} 0 & -\bar{B}_Z & \bar{B}_Y \\ \bar{B}_Z & 0 & -\bar{B}_X \\ -\bar{B}_Y & \bar{B}_X & 0 \end{bmatrix} \tag{24}$$

The above equation shows that matrix $\overline{\mathbf{B}}$ is a skew symmetric matrix composed by projective base line components. Similarly, for the matrix $\overline{\mathbf{R}}$, we may let its determinant equal to unit, namely

$$|\overline{\mathbf{R}}| = 1 \tag{25}$$

which shows that $\overline{\mathbf{R}}$ is a normalized matrix. Since Eq.(22) has the same form as Eq.(9), $\overline{\mathbf{R}}$ is equivalently called projective rotation matrix with the property of unit determinant.

As a summary of above derivation it is concluded: the fundamental matrix can be decomposed, up to a constant, as a product of the projective base line matrix $\overline{\mathbf{B}}$ and the projective rotation matrix $\overline{\mathbf{R}}$, where $\overline{\mathbf{B}}$ is a skew symmetric matrix formed by projective base line components and $\overline{\mathbf{R}}$ has unit determinant. In this way, model coordinates of a point in non-metric camera computer vision can be expressed in the same manner as in metric camera photogrammetry.

Once the fundamental matrix is obtained elements in matrices $\bar{\mathbf{B}}$ and $\overline{\mathbf{R}}$ can be calculated in the following way. First it should be pointed out the existence of the relation

$$\overline{\mathbf{E}}^T \bar{\mathbf{b}} = \mathbf{0} \tag{26}$$

From this homogeneous equation, the ratio of projective base line components $\bar{B}_Y / \bar{B}_X$ and $\bar{B}_Z / \bar{B}_X$ can be obtained. For the elements in the projective rotation matrix, we first write Eq.(22) according to its column

$$\bar{\mathbf{B}} \bar{\mathbf{r}}_i = \bar{\mathbf{e}}_i \quad (i=1,2,3) \tag{27}$$

Since $rank(\overline{\mathbf{B}}) = 2$, only two components in every vector $\bar{\mathbf{r}}_i$ (i=1,2,3) can be determined; or equivalently, six relations among the nine elements of the normalized matrix $\overline{\mathbf{R}}$ can be established. If the three elements in the first row of $\overline{\mathbf{R}}$ matrix are chosen as known, the other six elements can be expressed as

$$\begin{aligned}
\bar{r}_{21} &= (\overline{B}_Y \bar{r}_{11} + \bar{e}_{31}) / \overline{B}_X \qquad & \bar{r}_{31} &= (\overline{B}_Z \bar{r}_{11} - \bar{e}_{21}) / \overline{B}_X \\
\bar{r}_{22} &= (\overline{B}_Y \bar{r}_{12} + \bar{e}_{32}) / \overline{B}_X & \bar{r}_{32} &= (\overline{B}_Z \bar{r}_{12} - \bar{e}_{22}) / \overline{B}_X \\
\bar{r}_{23} &= (\overline{B}_Y \bar{r}_{13} + \bar{e}_{33}) / \overline{B}_X & \bar{r}_{33} &= (\overline{B}_Z \bar{r}_{13} - \bar{e}_{23}) / \overline{B}_X
\end{aligned} \tag{28}$$

Another primary quantity to be derived from the fundamental matrix is the ratio $k = \bar{\lambda}_2 / \bar{\lambda}_1$ - a cross ratio of lengths of two conjugate projective rays. Multiplying Eq.(18) with $\overline{\mathbf{B}}^T$ and noticing that $\overline{\mathbf{B}}^T \overline{\mathbf{b}} = \mathbf{0}$, the least squares solution to k can thus be written as

$$k = \frac{\overline{\lambda}_2}{\overline{\lambda}_1} = \frac{(\overline{\mathbf{E}}\overline{\mathbf{x}}_2)^T (\overline{\mathbf{B}}\overline{\mathbf{x}}_1)}{(\overline{\mathbf{E}}\overline{\mathbf{x}}_2)^T (\overline{\mathbf{E}}\overline{\mathbf{x}}_2)} \tag{29}$$

Above discussion shows that the fundamental matrix can be expressed as a product of the projective base line matrix and projective rotation matrix. From the fundamental matrix, one can determine the ratios of base line components and six relations among the elements of the projective rotation matrix, as well as the cross ratio of lengths of two conjugate projective rays. These are all the primary quantities derivable from the fundamental matrix of a stereo pair.

4. OBJECT RECONSTRUCTION

In this section, the homogeneous coordinates of a model point are derived and then a 3-D transformation is proposed to calculate its object coordinates.

1. Projective model and its computation

Let $\overline{\mathbf{u}}$ be the homogeneous coordinates of an object point. Its transformation to model coordinates $\overline{\mathbf{p}}$ is

$$\overline{\mathbf{p}} = \overline{\mathbf{T}}\, \overline{\mathbf{u}} \tag{30}$$

where $\overline{\mathbf{T}}$ is a 3*4 transformation matrix. Substituting Eq.(30) into Eq.(18) yields

$$\begin{aligned}
\overline{\mathbf{T}}\overline{\mathbf{u}} &= \bar{\lambda}_1 \overline{\mathbf{x}}_1 \\
\overline{\mathbf{B}}\overline{\mathbf{T}}\overline{\mathbf{u}} &= \overline{\lambda}_2 \overline{\mathbf{E}}\overline{\mathbf{x}}_2
\end{aligned} \tag{31}$$

There are six equations in total in Eq.(31). Any four of them will constitute a projective transformation of the object, provided its transformation matrix is not rank deficient. For symmetry, we choose the first two equations from the first group and last two equations from the second group in Eq.(31). Thus formed equation is

$$\mathbf{A}\overline{\mathbf{u}} = \bar{\lambda}_1 \bar{\mathbf{h}} \tag{32}$$

where

$$\bar{\mathbf{h}} = (\bar{h}_1\ \bar{h}_2\ \bar{h}_3\ \bar{h}_4)^T \tag{33}$$

is the homogeneous coordinates of the model point and can be calculated with

$$\begin{aligned}
\bar{h}_1 &= \bar{x}_1 \\
\bar{h}_2 &= \bar{y}_1 \\
\bar{h}_3 &= k(\bar{e}_{21}\bar{x}_2 + \bar{e}_{22}\bar{y}_2 + \bar{e}_{23}) \\
\bar{h}_4 &= k(\bar{e}_{31}\bar{x}_2 + \bar{e}_{32}\bar{y}_2 + \bar{e}_{33})
\end{aligned} \tag{34}$$

Comparison of Eq.(32) with Eq.(2) shows $\bar{\mathbf{h}}$ is actually a projective transformation of object $\bar{\mathbf{u}}$, and the $\bar{\mathbf{h}}$ vector for all points will then form a projective model of the object.

The projective coordinates can thus be calculated with

$$C(1,2,3,4,5,i) = \frac{|\bar{h}_1\ \ \bar{h}_2\ \ \bar{h}_3\ \ \bar{h}_4\,|\,|\bar{h}_1\ \ \bar{h}_2\ \ \bar{h}_5\ \ \bar{h}_i|}{|\bar{h}_1\ \ \bar{h}_2\ \ \bar{h}_3\ \ \bar{h}_5\,|\,|\bar{h}_1\ \ \bar{h}_2\ \ \bar{h}_4\ \ \bar{h}_i|} \tag{35}$$

Therefore, as soon as the cross ratio k is obtained from the decomposition of the fundamental matrix, the projective model can be reconstructed by calculating its homogeneous coordinates or projective coordinates with Eq.(34) or Eq.(35) respectively.

2. 3-D direct linear transformation (3D-DLT)

The 2-D DLT algorithm was proposed almost three decades ago [Abdel-Aziz et al, 1971]. It is used to establish the relationship between a 3-D object and its 2-D photograph whose coordinates are measured on a comparator with arbitrary orientation. The 3-D DLT algorithm to be proposed in this section will establish the relationship between a 3-D object and its 3-D projective model. The solution is linear.

Rewrite Eq.(32) with its elements,

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} U \\ V \\ W \\ 1 \end{bmatrix} = \bar{\lambda}_1 \begin{bmatrix} \bar{h}_1 \\ \bar{h}_2 \\ \bar{h}_3 \\ \bar{h}_4 \end{bmatrix} \tag{36}$$

Eliminate factor $\bar{\lambda}_1$ and denote a_{11} / a_{44} as L_{11} ...., then we obtain

$$\begin{aligned}
\frac{L_1U + L_2V + L_3W + L_4}{L_{13}U + L_{14}V + L_{15}W + 1} &= \frac{\bar{h}_1}{\bar{h}_4} \\
\frac{L_5U + L_6V + L_7W + L_8}{L_{13}U + L_{14}V + L_{15}W + 1} &= \frac{\bar{h}_2}{\bar{h}_4} \\
\frac{L_9U + L_{10}V + L_{11}W + L_{12}}{L_{13}U + L_{14}V + L_{15}W + 1} &= \frac{\bar{h}_3}{\bar{h}_4}
\end{aligned} \tag{37}$$

Eq.(37) is the 3-D DLT solution which takes a similar form as the 2-D DLT solution. In this way the well-known 2-D DLT algorithm has been generalized to 3-D DLT, which describes the relation between object and its projective model instead of its projective photograph as in 2-D DLT.

The algorithm for 2-D DLT solution can be used for 3-D DLT. After the homogeneous model coordinates are obtained with Eq.(34), the 3-D DLT solution is implemented. First, the unknown coefficients $L_1, L_2, \ldots L_{15}$ are determined linearly with minimum five known points by using least squares adjustment. The object coordinates $(U\,V\,W)$ will then be obtained by treating the $L_1, L_2, \ldots L_{15}$ coefficients as known parameters in Eq.(37). The object reconstruction is completed when all object points are determined.

5. TEST AND ANALYSES

The first step in projective reconstruction is to determine the fundamental matrix. One critical issue in this step is to ensure to robustness of the solution. Many efforts have been made from scientists in computer vision and photogrammetrists [Barakat, et al, 1997; Csurka et al, 1997; Deriche, et al, 1994; Luong, et al, 1994]. It is shown that the introduction of constraint Eq.(16) can stabilize the solution, especially when limited number of conjugate points are available in a stereo pair. However, the effect of this constraint is limited when a great number of points (>30) is involved in the computation [Shan, 1996]. Another issue in the implementation of the algorithm is the choice of the constant factor for the fundamental matrix. In order to avoid null elements, the element $\bar{e}_{32}$ is chosen as unit. The reason is explained as follows. When the photo interior orientation is done, $\bar{e}_{32}$ is approximately equal to $\overline{B}_X \bar{r}_{22}$ and the right term in Eq.(14) is $-\bar{y}_1$, which is corespondent to the y-parallax in metric camera photogrammetry [Shan, 1996]. In this way we further establish the parallelism of photogrammetry and computer vision.

In the computation of cross ratio, any five points with good geometric configuration in a stereo pair can be chosen as basis provided that none four of them are coplanar. Cross rations are computed from known object points with Eq.(5) and from homogeneous model coordinates with Eq.(34). Their discrepancies (measured by RMSE, root mean square errors) are listed in Tab.1 for different photo orientations numbered with 0-4, where 0 indicates the photo orientation is completed. Comparison of row 1,2,3,4 with row 0 shows that the results are consistent and therefore the discrepancies are due to image measurement errors. However, one should note that singularity occurs when an object point is coplanar with three of basis points. In this case, the 3-D projective transformation degenerates locally to a 2-D one. Thus, the cross-ratio of triangular areas should be used instead.

Tab.1 RMSE of cross ratios

Test Number	Cross ratios RMSE		
	σ_{C1}	σ_{C2}	σ_{C3}
0	0.172	0.110	0.170
1	0.172	0.057	0.177
2	0.172	0.052	0.170
3	0.170	0.044	0.175
4	0.173	0.063	0.181

Tab.2 RMSE of object points

Test Number	Coordinates RMSE		
	σ_X	σ_Y	σ_Z
2-D DLT	1.95.	1.58	1.74
1	1.93	1.46	1.72
2	1.91	1.47	1.72
3	1.92	1.47	1.75
4	1.91	1.49	1.74

For object reconstruction, a comparison study is designed between 2-D DLT and 3-D DLT solutions. In the 2-D DLT solution, six object points are selected as know points and the calculation is conducted for the image coordinates of each photo in the stereo pair. On the contrary, the 3-D DLT solution first constructs a projective model by calculating the fundamental matrix and then computing the homogenous coordinates $\overline{\mathbf{h}}$. The object is afterwards reconstructed by using the 3-D DLT solution given in Eq.(37). The 3-D DLT has been conducted for various photo orientations, numbered with 1 to 4 in Tab.2. Test results in Table 2 show that the results of those two solutions relative to known best values are very consistent and their discrepancies are within the tolerance of image measurement errors. Due to the utilization of the intrinsic relationship of a stereo pair, 3-D DLT can reach slightly better results than the 2-D DLT solution.

6. CONCLUDING REMARKS

The concluding remarks of this article can be summarized in following aspects. The cross ratio of volume elements is invariant in 3-D projective transformation. It can then be defined as projective-invariant coordinates to describe the object. There exist parallel concepts in photogrammetry with metric camera and computer vision with non-metric camera. It has been proven that the fundamental matrix can be expressed as a product of the projective base line matrix and the projective rotation matrix. Once the fundamental matrix is obtained from sufficient number of conjugate image points in a stereo pair, its base line components can be determined up to a constant, and six relationships among the nine elements of the projective rotation matrix can be determined too. One essential quantity to construct the projective model is the cross ratio k of lengths of two conjugate projective rays. The homogeneous coordinates of the projective model can be calculated with the fundamental matrix and its derivatives: cross ratio k and the projective base line matrix. The proposed 3-D DLT solution generalizes the existing 2-D DLT solution and reconstructs the object from its projective model. It needs minimum five known conjugate object points. If more than five are available, the least squares adjustment will be conducted for all points. The 3-D DLT can gain the same accuracy as the 2-D DLT solution, which should be applied to two photographs of a stereo pair with minimum six known points on each. Tests conducted in this article verify the derivation and validate the analyses.

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