A Photogrammetric Method for Single Image Orientation and Measurement

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Abstract
The aim of this paper is to present a photogrammetric method for determining the dimensions of flat surfaces, such as billboards, based on a single digital image. A mathematical model was adapted to generate linear equations for vertical and horizontal lines in the object space. These lines are identified and measured in the image and the rotation matrix is computed using an indirect method. The distance between the camera and the surface is measured using a lasermeter, providing the coordinates of the camera perspective center. Eccentricity of the lasermeter center related to the camera perspective center is modeled by three translations, which are computed using a calibration procedure. Some experiments were performed to test the proposed method and the achieved results are within a relative error of about 1 percent in areas and distances in the object space. This accuracy fulfills the requirements of the intended applications.

Introduction
One of the main advantages of photogrammetry is its ability for remotely taking measurements with no contact with the object. Several measurement tasks that are currently being performed by direct methods could benefit from advantages of the photogrammetric techniques if some restrictions were removed. In general, the use of stereopairs taken from dedicated photogrammetric cameras and software represents a barrier to the widespread use of this technique due to the high costs of the systems and the need for skilled photogrammetrists to achieve reliable results.

One example of the need for a low cost application is the recording and measurement of placards and billboards for advertising, which is of interest for local government authorities. An alternative photogrammetric method for digital recording and non-contact measurement of the advertisements areas could be useful, if the requirements are fulfilled:

- Low cost.
- Ease of field operation.
- Data processing should be performed by non-specialist users.
- Non-contact measurements (no control points).
- Full digital data-flow.
- Accuracy of 1 percent for areas is acceptable.

Considering that most of these placards and billboards are non-curved objects, a single image photogrammetric method appears to be the most suitable approach. Therefore, a method in two stages for image orientation was adapted from existing methods. In the first stage, image rotations (Euler angles) are computed using vertical and horizontal straight lines; and in the second stage, the perspective center (PC) coordinates are determined from a direct distance measurement using a lasermeter, avoiding any contact with the object.

Using Lines in Photogrammetry
Image orientation is a topic of interest for photogrammetrists, as well as for machine vision scientists. Interior and exterior orientation parameters that model the camera geometry and the relationship between the camera and the object reference systems must be computed using features previously matched in both spaces.

Line-based orientation of digital images has gained great interest, basically due to the potential of automation and the robustness of some methods of line detection. The method of plumb line calibration (Brown, 1971) was one of the earliest approaches using lines in photogrammetry. This method is suitable to recover the radial and decentering lens distortion parameters, while the remaining interior (focal length and principal point coordinates) and exterior orientation parameters have to be determined by a complimentary method.

The classical single photo resection (SPR) method solves the problem of image orientation using control points, whose coordinates are known both in the image and object reference systems. The non-linearity of the model and the problems related to target location in digital images are the main drawbacks of such classical approaches.

The use of digital geometric entities, such as ground control, was first presented by Lugnani (1980) and Masry (1981). The method of space resection proposed by Lugnani (1980) was based on an iterative improvement of the initial estimates, considering the projected features from the object to the image space, using the collinearity model and their discrepancies to the observed image features, which should be minimized. The treatment of lines under perspective projection was also considered by the computer vision community, especially with Haralick (1980) and Barnard (1983), who presented the concept of interpretation plane which encompasses a line in the image and in the object space.

Ethrog (1984) presented a photogrammetric method for determining the interior orientation parameters and the orientation angles using objects with parallel and perpendicular lines in reference to photographs taken with non-metric cameras.

There has been a significant number of papers using lines: Tommaselli and Lugnani (1988), Mulawa and Mikhail

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(1988), Haralick (1989), Dhome *et al.* (1989), Chen *et al.* (1989), Salari and Jong (1990), Liu *et al.* (1990), Wang and Tsai (1990), Lee *et al.* (1990), Echigo (1990), Chen and Jiang (1991), Chen and Tsai (1991), Tommaselli and Tozzi (1996), Quan and Kanade (1997), Van den Heuvel (1998), Habib (1999), Habib *et al.* (2003).

Haralick (1989) presented a method to compute the camera orientation parameters from the perspective projection of a rectangle in which the image scale remains unknown, unless a distance in the object space could be introduced. Lee *et al.* (1990) presented a similar approach using rectangular shapes for robot location.

Van den Heuvel (1998) also studied the use of a single image with geometric constraints for object reconstruction. Those constraints are based on geometric relations between straight lines, such as coplanarity, parallelism, perpendicularity, symmetry, and distance. The authors also approached the problem of camera orientation using parallel lines and the concept of vanishing points (Van den Heuvel, 1997).

More often, in the reviewed approaches, the problem of image scale is neglected or its determination requires a distance measured over the object. Since the objects to be measured are often unreachable, measuring the distance from the camera to the object with a lasermeter is a reasonable alternative to provide the camera position and image scale, while the rotation angles could be computed from vertical and horizontal straight lines.

Orientation Using Straight Lines

Straight Line Geometry in the Image Space

An interpretation plane can be defined as being the plane that contains the straight line in the object space, the projected straight line in the image space, and the perspective center of the camera (see Figure 1).

The normal vector N to the interpretation plane in the image space can be stated as a function of straight line parameters in the image space:

$$\mathrm{N} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} f \cdot \cos\theta \\ f \cdot \sin\theta \\ -\rho \end{bmatrix} \tag{1}$$

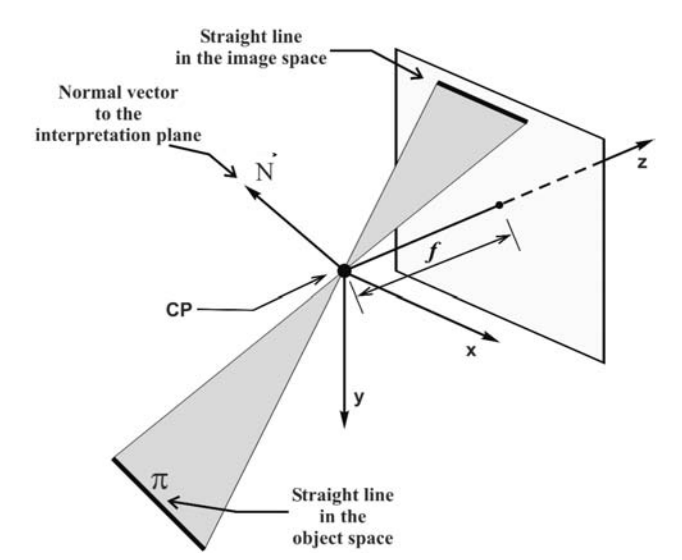

Figure 1. The interpretation plane and the normal vectors.

where f is the camera focal length; θ and ρ the elements of the normal representation of a straight line in the image space.

A Two-stage Approach for Image Orientation

Liu *et al.* (1990) proposed a mathematical model to estimate rigid body motion using straight lines. This approach is more advantageous in comparison with the simultaneous methods because it is a two-stage approach: rotations are computed in the first stage and translations in the second (Liu and Huang, 1988).

In the first stage, rotations are computed from the following condition equation:

$$n^T \cdot R^T \cdot \mathrm{N} = 0 \tag{2}$$

where n is the direction vector of the straight lines in the object space; N is the normal vector to the interpretation plane in the image space and R is the rotation matrix, defined by three sequential rotations ($R_\kappa R_\varphi R_\omega$). The rotated normal vector to the interpretation plane in the image space is perpendicular to the direction vector of the homologous straight line in the object space. Considering a non-linear solution with the orientation angles as unknown, at least three straight lines correspondences are required.

Equation 2 can be rewritten as:

$$[a \quad b \quad c] \cdot \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}^T \cdot \begin{bmatrix} A \\ B \\ C \end{bmatrix} = 0. \tag{3}$$

As just the direction vectors of the straight lines are considered at this stage, this model is suitable to compute orientation angles from vertical and horizontal lines, with the translations remaining to be computed in the second stage. In the proposed approach, the translations are computed from the camera to the object distance, which are directly measured during image acquisition by using a lasermeter; the X and Y coordinates can be set arbitrarily.

Computing Orientation Angles from Horizontal and Vertical Straight Lines

The parametric equation of a straight line in the object space is given by:

$$\begin{cases} x = x_1 + t \cdot a \\ y = y_1 + t \cdot b \\ z = z_1 + t \cdot c \end{cases} \tag{4}$$

with a, b, c being the components of the direction vector, and t a free parameter.

Considering the direction vectors for horizontal and vertical edges in the object space, two equations are easily obtained: Equation 5 for a horizontal line and Equation 6 for a vertical line.

$$F_H : f \cdot \cos\theta \cdot r_{11} + f \cdot \sin\theta \cdot r_{21} - \rho \cdot r_{31} = 0 \tag{5}$$

$$F_V : f \cdot \cos\theta \cdot r_{12} + f \cdot \sin\theta \cdot r_{22} - \rho \cdot r_{32} = 0. \tag{6}$$

At least three non-parallel straight lines are required to compute the orientation angles κ, φ and ω. If more than three straight lines correspondences are available, an estimation method can be used to get an optimum estimate for the orientation parameters. The least squares adjustment method with conditions (Mikhail and Ackerman, 1976) is suitable to estimate parameters from the functional model as presented in Equations 5 and 6 and from the observations obtained from the image.

Straight lines can be measured manually in the digital image from their endpoints coordinates using any ordinary image processing software or automatically by applying feature extraction methods.

Perspective Center Determination

In the second stage of the Liu and Huang (1988) approach, translations are indirectly computed using straight lines. This procedure can be avoided if the distance from the camera to the object is directly measured using a lasermeter, as proposed in this paper.

Let us consider the lasermeter reference system coincident with the photogrammetric system of the camera. This assumption is made to allow the derivation of an expression relating the measured distance with the perspective center coordinates in an arbitrary reference system. The existing displacements of the lasermeter origin with respect to the camera will be discussed in the next section.

The 3D coordinates of a point P in the object space can be transformed to the lasermeter reference system using a rigid body transformation, considering three rotations and three translations (Equation 7).

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R \cdot \begin{bmatrix} X - X_{PC} \\ Y - Y_{PC} \\ Z - Z_{PC} \end{bmatrix} \tag{7}$$

where: X_{PC}, Y_{PC}, Z_{PC} are the perspective center coordinates of the camera with respect to the object space reference system; X, Y, Z are the coordinates of a point P in the object space reference system; x, y, and z are the P coordinates in the camera reference system (or lasermeter, if they are considered coincident).

Considering that the straight lines lie onto a flat surface, the Z coordinates of the object can be stated as zero. The origin of the object space reference system can be freely translated, preserving areas and distances on the object (Figure 2). If it is assumed that the reference systems of the lasermeter and camera are coincident, the camera perspective center coordinates can be obtained using Equation 8:

$$\begin{bmatrix} X_{PC} \\ Y_{PC} \\ Z_{PC} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{21} & r_{31} \\ r_{12} & r_{22} & r_{32} \\ r_{13} & r_{23} & r_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ d \end{bmatrix} = \begin{bmatrix} r_{31}d \\ r_{32}d \\ r_{33}d \end{bmatrix} \tag{8}$$

where d is the distance between the camera and the object surface, measured with the lasermeter. The point P, which corresponds to the projection of the laser beam on the object plane, was considered as the origin of the object space coordinates (see Figure 2b).

Once the rotation matrix is computed by using vertical and horizontal straight lines, point coordinates in the object reference system could be determined using the inverse collinearity equations.

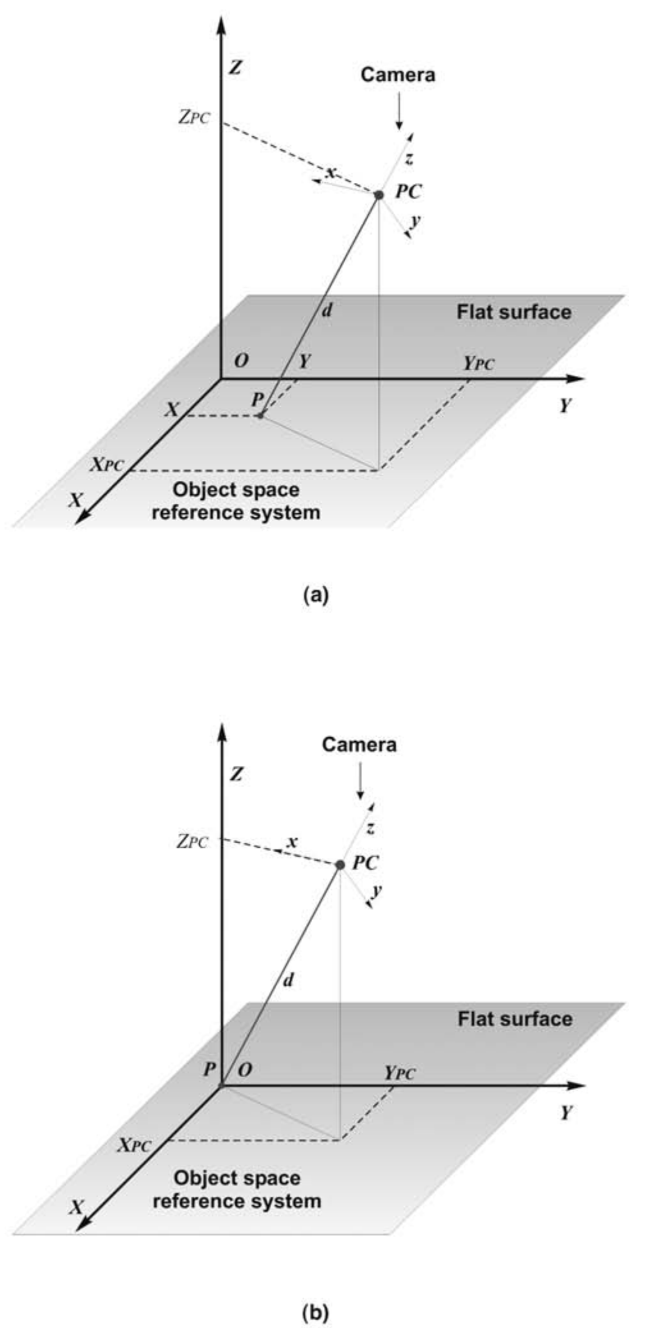

Figure 2. (a) Perspective center position related to the object space; (b) the origin of the object space reference system can be arbitrarily translated.

Modeling the Displacement Between the Camera and the Lasermeter

In the previous section, it was assumed that there was perfect coincidence between the camera and the lasermeter axis, a condition that is unfeasible to achieve in practice. As a consequence, the measured distance does not correspond to the distance between the camera perspective center and the point of intersection (A_C in Figure 3). The camera optical axis and the laser beam can be aligned mechanically, and then the remaining three translations between the two reference systems can be determined. The misalignment of the camera and lasermeter axis can be evaluated and corrected mechanically by taking several images of the projected beam over a plane parallel to the image frame, from different distances.

These eccentricity parameters (three translations) can be roughly measured with a scale over the prototype or through an indirect process. Figure 3 depicts the geometry of the camera and the lasermeter arrangement, with the eccentricity parameters. In general, the eccentricity in Y can be minimized if the camera and lasermeter axis are mechanically aligned in the x camera direction.

In Figure 3, PC and FC are the camera perspective center and the lasermeter center, respectively; A_C is the point of intersection between the camera optical axis and the XY plane, where the straight lines are located; A_T is the point of intersection between the laser beam and the same XY plane; D_C is the distance from the camera perspective center to the XY plane; D_T is the distance from the lasermeter center to the XY plane;

Using the inverse collinearity equations, the coordinates of the perspective center can be projected to the point A_c

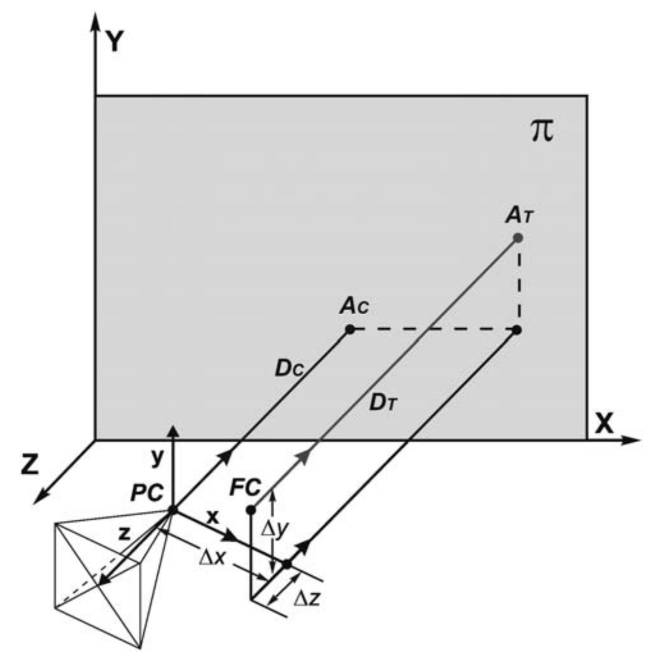

Figure 3. Geometry of the camera-lasermeter mount and the eccentricity.

onto the plane π, considering that $x = y = 0$ and $Z_{AC} = 0$, leading to Equation 9:

$$\begin{bmatrix} X_{A_C} \\ Y_{A_C} \end{bmatrix} = \begin{bmatrix} X_{PC} - Z_{PC} \cdot \dfrac{r_{31}}{r_{33}} \\ Y_{PC} - Z_{PC} \cdot \dfrac{r_{32}}{r_{33}} \end{bmatrix}. \tag{9}$$

Taking the plane π as a reference, and after some algebraic manipulations, distance D_C can be calculated using Equation 10:

$$D_C = \frac{Z_{PC}}{r_{33}}. \tag{10}$$

A similar equation can be established for the measured distance with the lasermeter:

$$D_T = \frac{Z_{FC}}{r_{33}}. \tag{11}$$

Let us now consider a rigid body transformation from the camera reference system to the lasermeter system. The coordinates of the lasermeter center can be obtained from the coordinates of the camera perspective center by:

$$\begin{bmatrix} X_{FC} \\ Y_{FC} \\ Z_{FC} \end{bmatrix} = \begin{bmatrix} X_{PC} \\ Y_{PC} \\ Z_{PC} \end{bmatrix} + R^T \cdot \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}. \tag{12}$$

Taking Z_{FC} from Equation 12 and combining with Equation 11 leads to:

$$Z_{PC} = D_T \cdot r_{33} - (r_{13} \cdot \Delta x + r_{23} \cdot \Delta y + r_{33} \cdot \Delta z). \tag{13}$$

Using Equation 13 the camera position, Z_{PC} can be calculated from the measured distance D_T, considering that the eccentricity was previously calibrated and the orientation matrix was determined from vertical and horizontal straight lines. The X_{PC} and Y_{PC} coordinates of the camera perspective center can be arbitrarily established.

The same model (Equation 13) can be used to compute indirectly the eccentricity parameters, since the camera position and orientation are known and that several images taken from different points of view are available. Considering, for simplicity, that the rotation matrix elements and the measured distance are free of errors, and that Z_{PC} is the observed quantity in Equation 13, the eccentricity parameters (Δx, Δy, and Δz) can be estimated using the least squares adjustment of indirect observations. Taking these assumptions into account, the model is linear, and at least four observations are required to estimate the parameters. If the rotation elements and the distance were considered as random variables, then, a more general method of adjustment of observations and parameters should be used (Mikhail and Ackerman, 1976).

The process of eccentricity calibration is carried out in two steps: the camera calibration and the eccentricity calibration itself. Several images of a calibration plate with known signalized points are acquired. For each one of these images a distance between the prototype, and a control point is measured with the lasermeter. The interior and exterior orientation parameters, and the adjusted values for the coordinates of points in the object space are computed using a self-calibrating bundle adjustment. The Z coordinate of the perspective center, obtained in the camera calibration step and the distance measured with the lasermeter, are used for the eccentricity calibration.

Experiments

System Calibration

A digital camera Kodak DX3500 was rigidly attached to a lasermeter Leica DISTO™ Memo, using a metallic support, designed to maintain the parallelism between the laser beam and the camera optical axis.

The DX3500 Digital camera has an array of 1800 × 1200 pixels, with a virtual pixel size of 0.0194 mm and a nominal focal length of approximately 38 mm, similar to a 35 mm equivalent frame. The lasermeter has a nominal accuracy of ±3 mm. Nine images with 54 control points were used in a self-calibrating bundle adjustment, considering interior orientation parameters as block invariant. Several groups of interior parameters were tested, and the most reliable results were used, considering the following parameters: f, x_o, y_o, K_1, P_1, P_2. The estimated standard deviations of these interior orientation parameters were considered to be within the expected tolerance. The computed *a posteriori* variance factor was tested against the *a priori* value using a χ^2 test and accepted with a level of significance of 5 percent.

Using the same nine images taken for the camera calibration, the eccentricity parameters were computed according to the model presented in the previous sections and the results are presented in the Table 1. The computed values for the eccentricity parameters are similar to the directly measured values, and the estimated standard deviations are compatible with the accuracy of the lasermeter (±3 mm) and the standard deviation in the Z coordinate of the perspective center obtained during the camera calibration (±1.5 mm). The computed *a posteriori* variance factor was tested against the *a priori* value using a χ^2 test and accepted with a level of significance of 5 percent.

Table 1. Eccentricity Parameters Between Camera and Lasermeter and Their Estimated Standard Deviations

Δx (mm)	$\sigma_{\Delta x}$ (mm)	Δy (mm)	$\sigma_{\Delta y}$ (mm)	Δz (mm)	$\sigma_{\Delta z}$ (mm)
−20	4	118	2	51	2

Assessment of the Methodology

In order to assess the accuracy of the whole system, the images taken during the calibration step and the calibrated eccentricity parameters were used in the experiments. These images were chosen because their exterior orientation parameters were already computed with a proven methodology (calibration with bundle adjustment) and with a high level of redundancy (48 control points).

The images were then oriented using straight lines, which were defined from two control points. The distances measured with the lasermeter and previously used for eccentricity calibration were also considered. The orientation angles computed with the adapted model were compared with those obtained in the calibration using bundle adjustment. Only the Z coordinate of the perspective center was compared because the X and Y components were obtained in an arbitrary reference system. Several distances and polygonal areas were defined linking control points and measured using an interactive software, implemented in C++ language. These measured elements were compared with their corresponding true values, which were computed from the coordinates of the control points. Several experiments were carried out varying the number and the configuration of straight lines used in the orientation step, but only the most representative are presented in this paper. The results of this experiment using 11 straight lines are presented in Table 2 and Table 3. The discrepancies in the rotations (Table 2) are compatible to their estimated standard deviations and the error in the Z coordinate is comparable with the nominal accuracy of the lasermeter (3 mm).

The accuracy of the system was tested by comparing measured areas and distances with their true values. It can be verified that a relative error less than 1 percent was achieved (Table 3), which can be considered suitable to the desired application. The main source of error is the uncertainty in the lasermeter measurement (±3 mm) which results in a scale error.

Besides the results presented in Tables 2 and 3, several other experiments were carried out with different configurations and parameters and the Root Mean Square Error of the relative discrepancies (RMSE%ε) for all the experiments were computed. The RMSE%ε for the measured areas were within 0.4 percent. For the distances, the results showed that the accuracy is better than 1 percent in the studied cases.

Some further conclusions can be derived from the experiments:

- The accuracy of the estimated rotations are dependent on the number and geometric configuration of the straight lines in the image, the camera inclination and the distance from the camera to the reference plane.
- In all the studied cases, the system returned values compatible with the estimated standard deviation and always within the initial expectation, i.e., errors smaller than 1 percent.

TABLE 2. DISCREPANCIES (ε) IN THE EXTERIOR ORIENTATION PARAMETERS OBTAINED WITH 11 STRAIGHT LINES

Rotations			Perspective Center
ε_κ	ε_φ	ε_ω	ε_Z
01′32″	−38″	−17′20″	2.5 mm

TABLE 3. RELATIVE DISCREPANCIES IN AREAS AND DISTANCES MEASURED WITH THE PROPOSED SYSTEM WHEN COMPARED TO THEIR KNOWN VALUES

Areas (%)			Distances (%)		
A_1	A_2	A_3	D_1	D_2	D_3
0.472	0.420	0.501	0.035	0.641	0.235

Figure 4. Example of placard showing distances used for checking the methodology (*le* = 975 mm).

Example the Application

The developed system was used to measure placards, building constructions and a historical facade. Just the first application will be presented in this paper, because it was the one for which the system was designed, according to the user's requirements. The main task in this kind of application is the measurement of distances, area and perimeters on the placard (Figure 4). Just one image was taken with the corresponding distance from the lasermeter to the plate (2.382 m). The image was then oriented using the existing vertical and horizontal straight lines and the measured distance. Some distances were directly measured on the placard, in order to check the final results (see Figure 4, distances *le, li, ce*, and *ci*). The same distances were measured in the image at three different positions and their corresponding ground values, computed with the proposed approach, were compared to those directly measured.

The RMSEε of the relative discrepancies in the distances was 0.389 percent, considering the distance from the camera to the object of 2.382 m. These results are equivalent to those obtained in the controlled experiment at a distance of 1.014 m. The RMSE of the relative discrepancies in the measured areas was 0.531 percent, which is also within the expected accuracy of the system. The relative discrepancies of the twelve measured distances are shown in Figure 5.

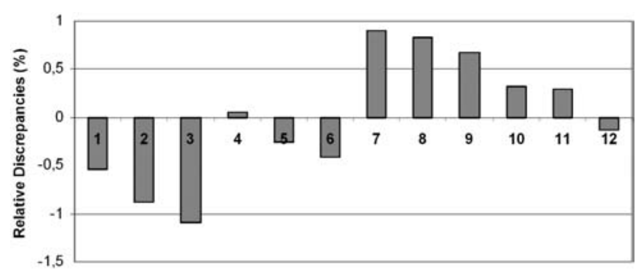

Figure 5. Relative discrepancies in the check distances measured on the placard.

Conclusions

This research evidenced that is feasible to employ single image photogrammetry for remote measurement of placards and other objects, meeting the user's requirements. The adapted mathematical model considering vertical and horizontal lines in the object space showed to be suitable to the proposed task. Using a lasermeter to measure the distance from the camera to the surface, no other source of external information is required to compute orientation, object coordinates and areas over the flat surface.

Experiments showed that areas and distances can be computed with relative errors around 0.5 percent, which can be considered suitable for several other applications, such as *as-built* surveying, restitution of facades, and measurement of transportation objects.

It is recommended to the field operator to maintain the camera and the lasermeter rigidly constrained together during image capture and distance measurement in order to guarantee the geometric conditions from which the models were derived.

Some recommendations for future work include:

- A combination of plumb line calibration method with single station data collection (Fryer, 1996) can improve the method; the camera could be calibrated on the job with the same imagery used to measure the object.
- To implement an algorithm for prediction and location of vertical and horizontal lines in the image, enabling automatic computation of the rotation angles.
- To study the integration of a low cost GPS receiver in order to collect georeferenced images, which will expand the horizons of applications of this methodology.
- To combine the proposed approach with two photo intersection.

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