A NEW ALGORITHM TO CORRECT FISH-EYE- AND STRONG WIDE-ANGLE-LENS-DISTORTION FROM SINGLE IMAGES

Christian Bräuer-Burchardt and Klaus Voss

Friedrich Schiller University Jena, Department of Computer Science, Digital Image Processing Group
D-07740 Jena, Germany
email: cbb@pandora.inf.uni-jena.de

ABSTRACT

The use of super-wide angle and fish-eye lenses causes strong distortions in the resulting images. A methodology for the correction of distortions in these cases using only single images and linearity of imaged objects is presented in this work. Contrary to most former algorithms, the algorithm discussed here does not depend on information about the real world co-ordinates of matching points. Moreover reference points determination and camera calibration is not required in this case.

The algorithm is based on circle fitting. It requires only the possibility of the extraction of distorted image points from straight lines in the 3D scene. Further, the actual distortion must approximately fit the chosen distortion model. For most fish-eye lenses appropriate distortion correction results can be obtained.

1. INTRODUCTION

It is not always possible to have distortion free camera systems. Therefore quantitative measurements are much sought in such cases. The commonly used wide-angle and fish-eye lenses contributes to image distortion owing to the optical properties of such lenses. When the use of such lenses is unavoidable, a correction methodology is required to do away with such distortions. Normally this is achieved by deriving the distortion function within the procedure of camera calibration.

Numerous recent publications dealt with the problem of camera calibration using distortion models, e.g. [2,4,5,9,12]. Methods of direct nonlinear minimization, closed-form solutions and two-step methods are discussed by [14]. Stein [11] as well as Sawhney [7] where the presented methods use point correspondences from multiple images. Shah and Aggarwal [8] gave a calibration technique for the intrinsic camera parameters of a fish-eye lens camera. Xiong and Turkowski [15] described a self-calibration method for fish-eye lenses. These techniques require substantial effort as much as camera calibration. However, in most cases, the process of camera calibration proves to be too costly or impossible to execute. An alternative simpler methodology is required in such situations.

A method using just the assumption of the linearity of lines and the information available from one single image was presented by Prescott and McLean [6]. However, the proposed estimation of straight line parameters by least squares linear regression is not robust enough in cases of very strong distortions resulting from fish-eye lenses.

The necessary assumption about linearity can generally be made using images of architectural objects. In the field of building reconstruction and architectural modeling there lies the necessity for the solving of reconstruction tasks with input images which is always obtained with expensive distortion free camera systems. So image distortions caused by the effects of radial lens distortion of wide angle-lenses are unavoidable.

Given an image with strong radial distortion (see Fig.1a) the first goal is to determine the parameters of the radial lens distortion, i.e. the distortion function and the distortion centre and, subsequently to correct the distortion effects.

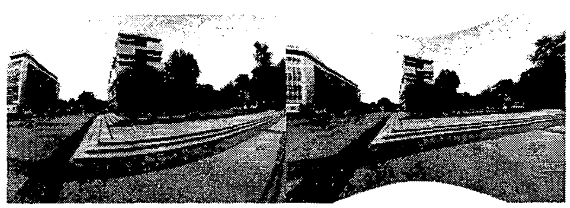

Fig.1. Original fish-eye- (left) and corrected (right) image

2. MODEL

In order to describe the 3D-2D mapping from the real world co-ordinates to the undistorted image co-ordinates we use the common pinhole camera model with isotropic pixel scaling and without shearing. Decentering distortion as described e.g. in [8,9,14] is neglected. The distortion centre is assumed to be equal to the principal point and situated in the intersection of the image plane with the optical axis of the camera system.

Let (x',y') be the measurable co-ordinates of the distorted image points, (x,y) the co-ordinates of the undistorted image points, $P=(X,Y)$ the principal point, r' the distance of the distorted points to P', and r the distance of the undistorted points to P. Since P is the distortion centre, we have $P= P'$. Close to the usual modeling the distortion function can be expressed as

$$\begin{aligned} r' &= f(r) = \frac{\sqrt{1+4cr^2}-1}{2cr} \\ r &= f^{-1}(r') = \frac{r'}{1-cr'^2}. \end{aligned} \tag{1}$$

Thus the image co-ordinates are obtained as

$$\begin{aligned} x' &= X + (x-X)\frac{\sqrt{1+4cr^2}-1}{2cr^2} \\ y' &= Y + (y-Y)\frac{\sqrt{1+4cr^2}-1}{2cr^2} \end{aligned} \tag{2}$$

$$x = X + \frac{x'-X}{1-cr'^2} \qquad y = Y + \frac{y'-Y}{1-cr'^2} \tag{3}$$

with $r^2 = (x-X)^2 + (y-Y)^2$ and $r'^2 = (x'-X)^2 + (y'-Y)^2$.

A negative value of c means that the undistorted image points are moved away from the distortion centre and a positive value of c implies a compression of the image. Now we consider the typical case of positive c which leads to so called barrel distortion.

Because $\lim_{r\to\infty} f(r) = \frac{1}{\sqrt{c}}$ all infinite points will be mapped onto a circle C_∞ with a radius $R = 1 / \sqrt{c}$ depending on the distortion coefficient c. Because of numerical reasons we will use the more descriptive quantity R as the distortion parameter as well as c. A straight line g in the undistorted image can be described by

$$(x - x_f) \cdot (X - x_f) + (y - y_f) \cdot (Y - y_f) = 0 \tag{4}$$

where (x_f, y_f) is the point on g through P perpendicular to g. After reformulation we get

$$(x-X)(X-x_f)+(y-Y)(Y-y_f)+(X-x_f)^2+(Y-y_f)^2 = 0. \tag{5}$$

Applying (3) to (5) and setting $u^2 = (X-x_f')^2 \cdot (Y-y_f')^2$ we get by

$$c\left((x'-X)^2+(y'-Y)^2\right)-(x'-X)\frac{x_f'-X}{u^2}-(y'-Y)\frac{y_f'-Y}{u^2}=1 \tag{6}$$

the equation of a circle in the general form. Thus any straight line g not intersecting P is transformed into a circle with some radius ρ and centre (ξ,η) by the radial lens distortion. Comparison of the coefficients yields

$$c = \frac{1}{\rho^2 - (\xi - X)^2 - (\eta - Y)^2} \tag{7}$$

and $$R^2 = \rho^2 - (\xi - X)^2 - (\eta - Y)^2. \tag{8}$$

3. ALGORITHM

In order to use our model we have to find points in the image whose undistorted image points are located on straight lines. Corresponding to the model the distorted points are on circle segments. An optimal circle is fitted to these points by least-square methods. If the principal point is known the distortion coefficient can be calculated from the circle data according to formula (8). However, the principal point is usually unknown and hence there may be errors in the circle data. So we search for a larger number of fitted circles resulting from straight line segments. The actual data of R and P will be determined by least-squares techniques.

3.1. Circle detection

To find a set of representative circles generated from distorted straight lines the following method is suggested. First, a number of edge segments approximating the periphery of the circle needs to be found. In order to find the edges as shown in Fig.2a we use an edge detection algorithm, e.g. [3]. To control the quality of edge detection one can look at the original image or the gradient image and restrict the allowed length of the segments.

In order to extract circles from the set of edge segments all pairs of segments were tested to determine whether they belong to the same circle. Assuming that the segments approximate the circle, the segments belonging to the same circle have the same distance to the centre of the circle. This test is performed by determining the intersection of the perpendicular bisectors of the two segments. If both segments have approximately the same distance to the intersection point they are candidates to belong to the same circle.

As a result of the algorithm a number of circles is obtained (see Fig.2b). To extract those that derive from radial distortion, every circle is weighted with the probability to be a "representative" one.

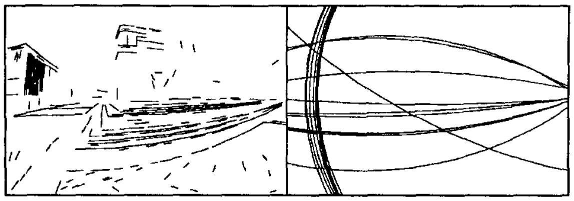

Fig.2. Extracted segments (left) and constructed circles (right)

3.2. Calculation of the distortion coefficient

Reformulating equation (8) we get

$$0 = -2\xi X - 2\eta Y + X^2 + Y^2 + R^2 + \xi^2 + \eta^2 - \rho^2 \quad (9)$$

Having n but at least three circles (ξ_i, η_i and ρ_i) the optimization task

$$S = \sum_{i=1}^{n} \left(\left(-2\xi_i\right) A + \left(-2\eta_i\right) B + C + \left(\xi_i^2 + \eta_i^2 - \rho_i^2\right) D \right)^2 \rightarrow \min \quad (10)$$

with

$$X = A / D$$
$$Y = B / D$$
$$R = \sqrt{\frac{C}{D} - X^2 - Y^2}$$

may be formulated. The task (10) can be solved by least squares methods as described in [13].

3.3. Distortion correction

After calculating distortion coefficient c the image has to be transformed by the inverse distortion function given by (3) to obtain the corrected image. Although the inverse function is explicit we use the forward distortion function (2) in an inverse manner in order to avoid digitization effects. For every pixel in the corrected image its original subpixel-exact position in the distorted image is determined and the grey value is obtained by bilinear interpolation (see Fig.1b).

4. RESULTS AND ACCURACY

The algorithm described was implemented to perform the correction of radial lens distortions without interaction with the user. The computation time for the whole procedure including the image restoration depends only on the image size. It takes approximately 1 sec on a pentium 500 PC to correct a 0.5 MB image.

4.1. Results on simulated data

In order to determine the accuracy the following experimentation was performed. In a 800 x 600 pixel image ten straight lines were randomly placed and at least ten points on every line were selected. The distortion coefficient had a fix value (R=700 and R=1600), but the distortion centre P was randomly chosen and placed not farther than 150 pixels from the image centre. The whole procedure was performed 100 times. The points were distorted according to P and R and disturbed in position by random noise with given standard deviation σ=0.0, 0.1, 0.2 and 0.5 pixels in x- and y-co-ordinate. The mean errors of $P=(X,Y)$ and R ($E_R=R_{calc}-R$, $E_X=X_{calc}-X$, $E_Y=Y_{calc}-Y$) and standard deviations (SD) (S_R, S_X, S_Y) were determined.

The results are shown in the tables 1 and 2, where n is the number of successful calculations (in the other 100–n cases the noise prevented a meaningful result).

Table 1: accuracy results (mean error and SD, in pixels), simulated data, strong distortion (R=700)

σ	$E_R \pm S_R$	$E_X \pm S_X$	$E_Y \pm S_Y$	n
0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	100
0.1	0.08 ± 0.50	0.03 ± 0.31	0.01 ± 0.31	100
0.2	-0.03 ± 1.04	0.09 ± 0.69	-0.04 ± 0.71	100
0.5	0.61 ± 2.97	0.33 ± 2.32	-0.18 ± 2.13	100

Table 2: accuracy results (mean error and SD, in pixels), simulated data, weak distortion (R=1600)

σ	$E_R \pm S_R$	$E_X \pm S_X$	$E_Y \pm S_Y$	n
0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	100
0.1	0.67 ± 6.66	-0.11 ± 2.35	0.09 ±2.02	100
0.2	2.60 ± 13.24	0.10 ± 5.88	0.21 ± 4.26	98
0.5	7.69 ± 24.78	1.14 ± 11.88	-0.73 ± 9.16	78

A value for σ=0.2 was experimentally found to be realistic for typical images. Assuming an error of e=±1.04 for R=700 this means a maximum error in the corners of the corrected image of ±0.55 pixels and ±0.73 pixels in the case of R=1600 using e=±13.24.

The same data were used as input of the method of Prescott and McLean [6] (PMM) using the known value of the distortion centre. This method is based on estimating the distortion and the straight line parameters by minimizing the distance of the corrected points to fitted straight lines. The results for both strong (R=700) and weak distortion (R=1600) are shown in table 3.

Table 3: accuracy results, simulated data, PMM

σ	$E_R \pm S_R$ (R=700)	n	$E_R \pm S_R$ (R=1600)	n
0.0	0.22 ± 0.90	100	1.81 ± 4.49	98
0.1	0.23 ± 1.18	100	0.04 ± 9.31	94
0.2	0.55 ± 2.19	100	2.57 ± 19.94	96
0.5	-0.08 ± 3.32	100	2.70 ± 22.12	76

4.2. Results on real images

We successfully corrected a number of highly distorted fish-eye and also wide-angle photographs. In some cases an interactive selection of the detected circles was necessary. In the case of proper single images it is difficult to estimate the mean error between the true values and the calculated values of the radial distortion coefficient and of the principal point because no information about the real undistorted data is available.

However, the reproducibility of the distortion coefficient and of the principal point was determined in experiment. We used a number of n=20 photographs taken by the same lens (Canon fish-eye EF 15/2.8) mounted on a usual SLR camera and digitized into 865 x 595 pixel images (see Fig.1). The calculated quantities R, X and Y were averaged over all analysed images. The results (mean ± standard deviation) were 446±15.0 (X), 295±6.2 (Y), and 632±9.7 (R), yielding coefficients of variation (CV) of 3.4% (X), 2.1% (Y), and 1.5% for R. This means a maximum error in the image corners of about ±7 pixels.

The same procedure was applied to a number of n=12 images of a grid pattern taken by video camera with 4.8 mm lens with weaker distortion (see Fig.3). The image size was 512 square and the results were 287±7.7 (X), 249±3.4 (Y), and 1338±20 (R). Hence the CV were 2.7% (X), 1.4% (Y), and 1.5% (R), yielding a maximum error in the image corners of approximately ±0.7 pixels.

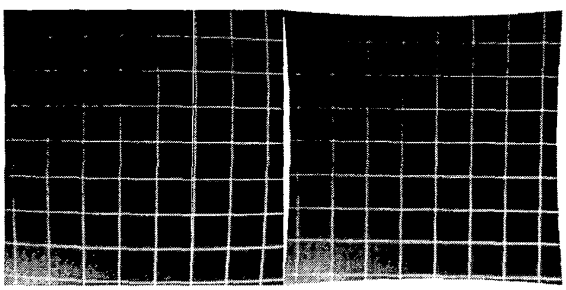

Fig.3. Distorted and corrected image of grid pattern

5. DISCUSSION AND SUMMARY

Although we analysed only the case of positive coefficients c the algorithm also works in the case of negative values of c. There are only certain restrictions that have to be reversed. However with regard to a strong radial distortion, this case is not the typical one.

The results of the correction depend on how well the chosen distortion model fits the actual lens distortion. Especially for strongly distorted fish-eye photographs the algorithm supplies acceptable results. In the case of strong distortions our algorithm is simpler, faster, and more robust than the method of Prescott and McLean [6]. The big advantage of our method is that a well distribution of the extracted image points over the whole image is not necessary. When extracting distorted points on long circle arcs, even very few circles are sufficient to calculate the correct distortion parameters. In the very extreme case only three circles each represented by just three points are sufficient to determine the strong lens distortion.

However, the distortion function must exactly fit the chosen one-parametric model. If two or more distortion coefficients are necessary a more accurate model as proposed e.g. in [1, 6, 9] has to be chosen. Our algorithm based on circle fitting, however, could be applied to give a first approximative solution.

6. REFERENCES

[1] C.Bräuer-Burchardt and K.Voss: Automatic lens distortion calibration using single views. In: Mustererkennung 2000, Springer, pp. 187-194, 2000

[2] D.C.Brown: Close-range camera calibration. Photogram.Eng. 37(8), pp. 855-866, 1971

[3] J.B.Burns, A.R.Hansen and E.MRiseman: Extracting straight lines. IEEE Trans PAMI (8), pp. 425-455, 1986

[4] F.Devernay and O.Faugeras: Automatic calibration and removal of distortion from scenes of structured environments. SPIE (2567), pp. 62-72, 1995

[5] Y.Nomura, M.Sagara, H.Naruse, and A.Ide: A simple calibration algorithm for high-distortion-lens camera. PAMI(14), No 11, pp. 1095-1099, 1992

[6] B.Prescott and G.F.McLean: Line-based correction of radial lens distortion. GMIP(59), No.1, pp. 39-47, 1997

[7] H.S.Sawhney and R.Kumar: True multi-image alignment and its application to mosaicing and lens distortion correction, IEEE Trans PAMI(14), No 3, pp. 235–243, 1999

[8] S.Shah and J.K.Aggarwal: Intrinsic parameter calibration procedure for a (high distortion) fish-eye lens camera with distortion model and accuracy estimation. PR(29), No.11, pp. 1775-1788, 1996

[9] S.W. Shih, Y.P. Hung, and W.S. Lin: When should we consider lens distortion in camera calibration. PR(28), No 3, pp. 447-461, 1995

[10] C.Steger: Evaluation of subpixel line and edge detection precition and accuracy. IAPRS, Volume XXXII, Part 3/1, pp. 256-264, 1998

[11] G.P.Stein: Lens distortion calibration using point correspondences. Proc CVPR, pp. 602-608, 1997

[12] R.Y. Tsai: An efficient and accurate camera calibration technique for 3-D machine vision. IEEE Proc CCVPR, pp. 364-374, 1986

[13] K.Voss, H.Suesse: Adaptive Modell und Invarianten für zweidimensionale Bilder. Shaker, Aachen, 22ff, 1995

[14] J. Weng, P. Cohen, and M. Herniou: Camera calibration with distortion models and accuracy evaluation. PAMI(14), No 11, pp. 965-980, 1992

[15] J. Xiong and K. Turkowski: Creating Image-based VR using a self-calibrating fisheye lens. Proc. CVPR '97